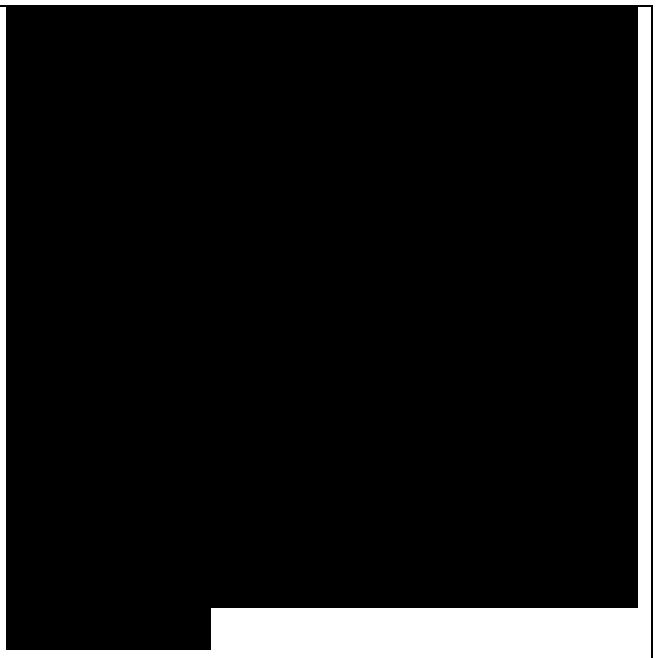
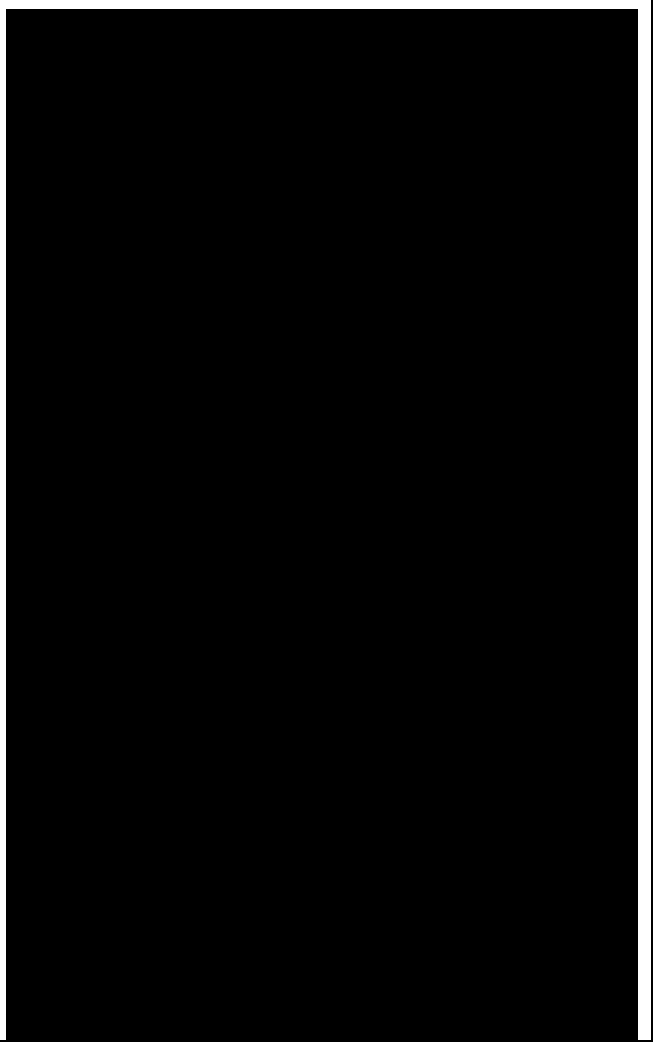
We propose a model of a nonlinear double-well potential NDWP, alias a double-well pseudopotential, with the objective to study an alternative implementation of the spontaneous symmetry breaking SSBin BoseEinstein condensates BECsand optical media, under the action of a potential with two symmetric minima. In the limit case when the NDWP structure is induced by the local nonlinearity coefficient	Chúng tôi đề xuất mô hình NDWP giếng thế kép phi tuyến, hay còn gọi là giả thế giếng kép nhằm tìm kiếm một phương pháp khác để thực thi phá vỡ đối xứng tự phát SSB trong trạng thái ngưng tụ BoseEinstein BEC và môi trường quang học, dưới tác động của thế có hai cực tiểu đối xứng. Trong trường hợp giới hạn khi cấu trúc NDWP được hình thành dưới tác động của hệ số phi tuyến cục bộ được biểu diễn qua tập hợp hai hàm delta,

represented by a set of two delta functions, a fully analytical solution is obtained for symmetric, antisymmetric, and asymmetric states. In this solvable model, the SSB bifurcation has a fully subcritical character. Numerical analysis, based on bothdirect simulations and computation of stability eigenvalues, demonstrates that, while the symmetric states are stable up to the SSB bifurcation point, both symmetric and emerging asymmetric states, as well as all antisymmetric ones, are unstable in the model with the delta functions. In the general model with a finite width of the nonlinear-potential wells, the asymmetric states quickly become stable, simultaneously with the switch of the SSB bifurcation from the subcritical to supercritical type. Antisymmetric solutions may also get stabilized in the NDWP structure of the general type, which gives rise to a bistability between them and asymmetric states. The symmetric states require a finite norm for their existence, an explanation to which is given. A full diagram for the existence and stability of the trapped states in the model is produced. Experimental observationof the predicted effects should be possible in BEC formed by several hundred atoms.	chúng ta sẽ thu được một nghiệm hoàn toàn giải tích đối với các trạng thái đối xứng, phản đối xứng và bất đối xứng. Trong mô hình có thể giải được này, điểm rẽ nhánh SSB có đặc trưng hoàn toàn chưa tới hạn. Phân tích số dựa trên cả mô phỏng trực tiếp và tính toán các trị riêng ổn định cho thấy, trong khi các trạng thái đối xứng ổn định đến điểm rẽ nhánh SSB, trạng thái đối xứng và không đối xứng mới nổi lên cũng như tất cả các trạng thái phản đối xứng không ổn định trong mô hình có các hàm delta. Trong mô hình tổng quát với độ rộng giếng thế phi tuyến hữu hạn, các trạng thái bất đối xứng nhanh chóng trở nên ổn định, đồng thời chuyển các điểm rẽ nhánh SSB từ loại dưới tới hạn sang siêu tới hạn. Các nghiệm phản đối xứng cũng có thể ổn định trong cấu trúc NDWP loại tổng quát, làm nảy sinh hiệu ứng lưỡng ổn giữa chúng và các trạng thái phản đối xứng. Các trạng thái đối xứng cần chuẩn hữu hạn cho sự tồn tại của chúng và chúng ta sẽ giải thích vấn đề này. Chúng ta cũng sẽ xây dựng giản đồ đầy đủ về sự tồn tại và tính ổn định của các trạng thái bẫy trong mô hình. Quan sát thực nghiệm các hiệu ứng dự đoán là khả thi trong BEC được hình thành bởi vài trăm nguyên tử.
Introduce 10 h 46 phút The one-dimensional 1DSchrödinger equation including a symmetric potential structure produces single-particle wave functions of a definite parity, even or odd, with theground state always corresponding to an even function without zeros. However, a spatially symmetric Hamiltonian of an interacting many-particle system can give rise to asymmetricstates, which may be	[redacted]

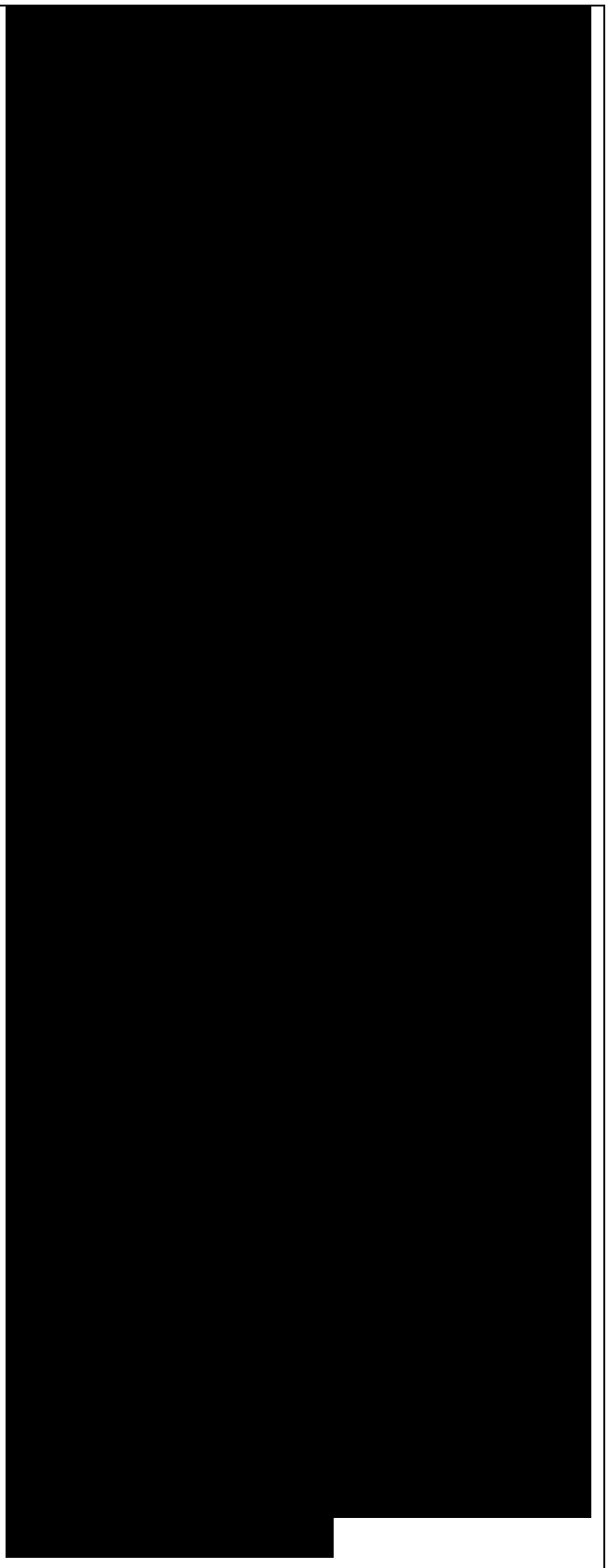
considered as a spontaneous-symmetrybreaking SSBeffect. At the classical level, the SSB occurs in optics, as a result of the interplay between the nonlinearity and waveguiding structures, when the strong nonlinearitypartly suppresses the linear coupling between parallel guiding cores. In particular, it was shown that a stable trapped mode may be asymmetric in a channel waveguide embeddedin the self-focusing Kerr medium 1. The onset of a sharpsymmetry-breaking instability in a double-hump twocomponent spatial optical soliton was demonstrated experimentally in a planar nonlinear waveguide 2.A natural setting in which SSB phenomena may arise inthe context of nonlinear optics and Bose-Einstein condensation BECis provided by double-well potentials DWPs. Inthe experiment, an effective optical DWP was created by aspecially designed illumination pattern applied, in the ordinary polarization, to a photorefractive crystal the SSB wasobserved in a beam with extraordinary polarization, shone through this structure 3. It was also proposed to realizesimilar effective potentials in coupled nonlinear microcavities 4, and in a structured core of an optical fiber 5. Aspecific variety of the optical SSB was studied in a model oftwo parallel-coupled antiwaveguides (ống dẫn sóng có độ chênh lệch chiết suất giữa lõi và vỏ âm) with the self-focusingnonlinearity, which corresponds to an effective doublebarrier potential, rather than DWP 6.Well-known dual-core optical fibers 7, which may serve as a basis for the power-controlled all-optical switching, ifthe Kerr nonlinearity is taken into regard 8, may also beconsidered as DWP structures, with

the difference that thetunneling between two potential wells is replaced by the linear coupling between the cores. In addition to the SSB ofcontinuous-wave states 9, the formation of asymmetricsolitons in dual-core fibers was studied in detail theoretically10. Similar analysis of the SSB for soliton modes was performed in models of dual-core fiber Bragg gratings with theKerr nonlinearity 11, and coupled waveguides with thequadratic 12and cubic-quintic 13nonlinear terms, including a system of linearly coupled complex GinzburgLandau equations of the cubic-quintic type 14. In Refs.15–17, the analysis of the SSB was extended to three-corelinearly coupled triangular configurations—for optical fibers, Bragg gratings, and complex Ginzburg-Landau equations, respectively.The concept of SSB also plays an important role in understanding experimental phenomena in BEC, because, if interactions between atoms are strong enough, the ground stateof the condensate may not follow the symmetry of the trapping potential 18. In particular, manifestations of SSB wereobserved in a quenched (bị dập tắt, làm lạnh) ferromagnetic state of a spinor three-componentcondensate 19. In the single-componentBEC, a natural setting for the realization of SSB may again be provided by DWP configurations. An effectively onedimensional DWP structure was realized experimentally inRef. 20. Loading a condensate of 87Rb atoms with the repulsive interaction between them into this structure made itpossible to observe Josephson oscillations for a small number of atoms, and the macroscopic quantum self-trapping featuring an imbalance between populations of the two wells, for a larger





number. Parallel to the experimental work, numerous theoretical studies of matter-wave DWP settings have been performed for the cases of both repulsion andattraction between atoms. These studies addressed problemssuch as finite-mode reductions 21 including twocomponent mixtures 22 , obtaining analytical results forspecific shapes of the potential 23, quantum effects 24,and some others. Recently investigated tunneling betweenvortex and antivortex states in BEC trapped in a twodimensional 2Danisotropic potential 25belongs to thiscategory too.Theoretical analysis was also performed for 2D and 3D extensions of the DWP settings in BEC, which add one ortwo extra dimensions to the model, either without an additional potential, or with a periodic optical-lattice OLpotential acting in these directions. These settings may be approximated, similar to the above-mentioned standard model ofdual-core optical fibers, by a system of linearly coupled 1D26or 2D 27equations. In a more accurate form, nearly1D solitons can be found as solutions to the full 2D equationthat includes the DWP the potential depends on the transverse coordinate x, allowing solitons to self-trap in the freelongitudinal direction y 28. The latter model is relevant tothe case of the self-attractive nonlinearity. In the case ofself-repulsion, dual-core gap solitons have been predicted in the setting with the OL potential applied along direction y29. Note that, in any setting, gap solitons cannot realize theground state of the respective system, but, nevertheless, theyrepresent stable configurations that have been created in theexperiment using the condensate of 87Rb atoms with the

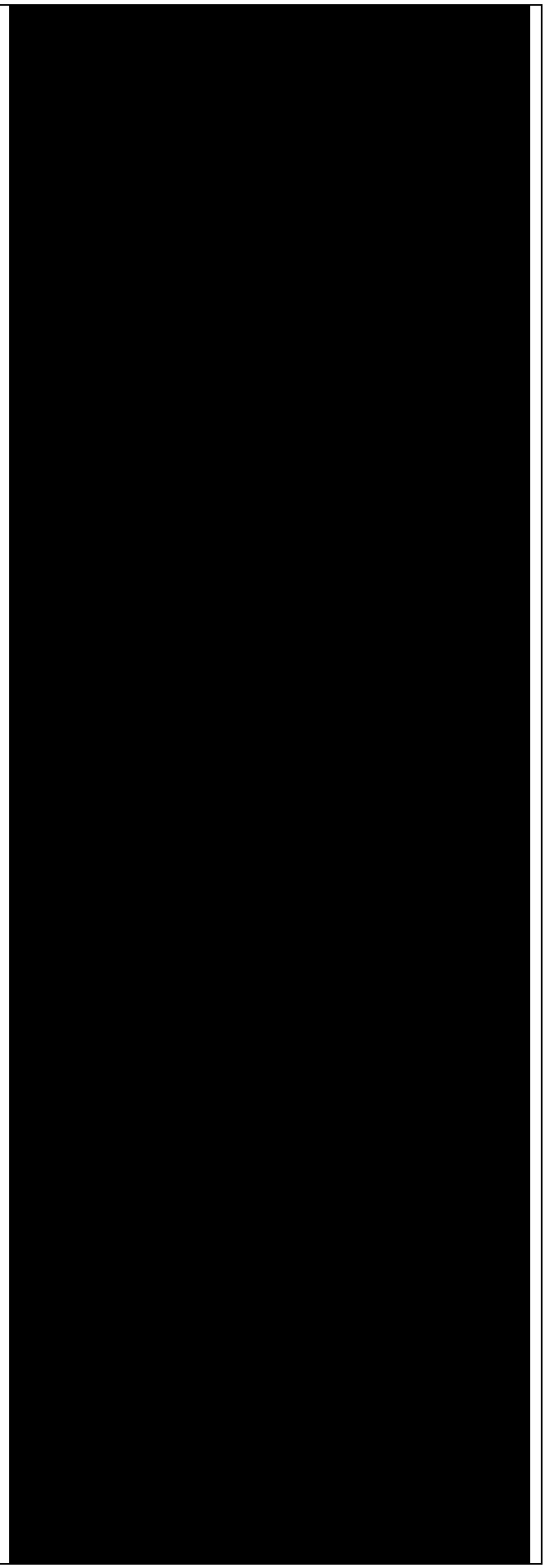


repulsion between them 30.

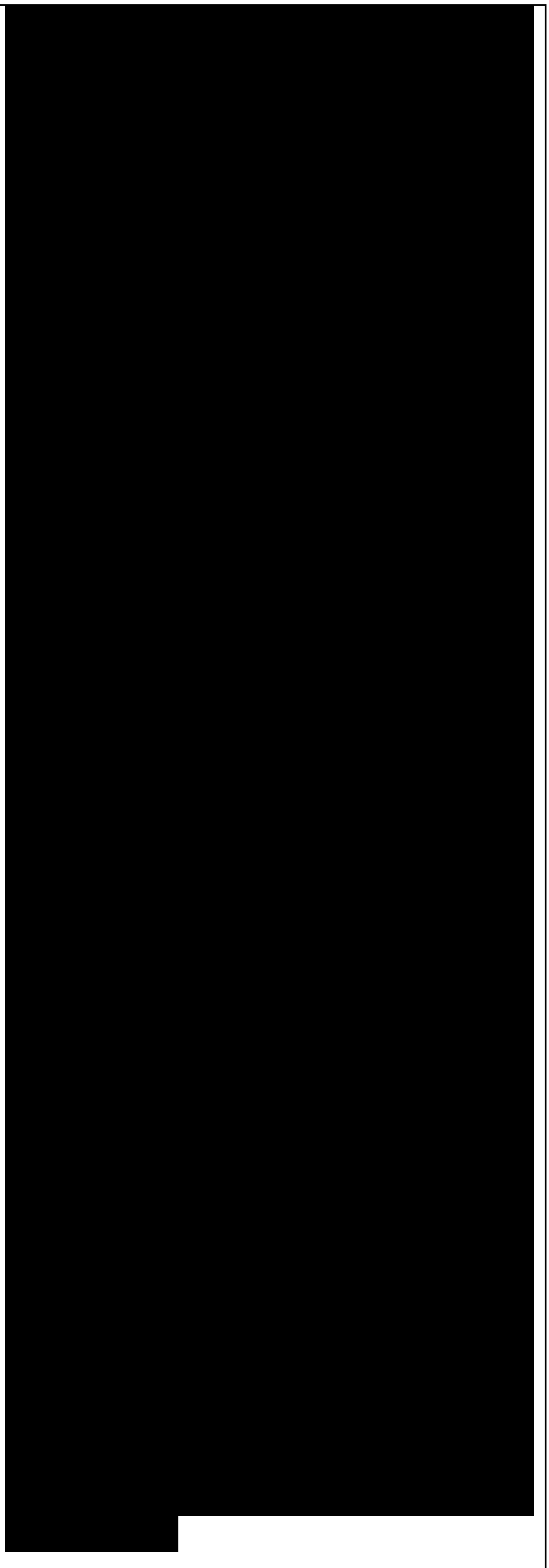
~~A general principle, upheld by the analysis in various settings in nonlinear~~ optics and BEC alike, is that the SSB occurs through bifurcations of symmetric or antisymmetricstates, in the models with the self-attraction and selfrepulsion, respectively. As mentioned above, models of theDWP or double-core type, combining cubic attractive andquintic repulsive nonlinearities, were studied too13,14,31,32. In the latter case, the competition between theself-focusing and self-defocusing against the backdrop of theDWP structure gives rise to specific SSB bifurcation diagrams, in the form of nonconvex closed loops 13,32, aswell as to specific dynamical switching regimes 31. Alsopredicted were manifestations of the SSB in a twocomponent BEC mixture trapped in the DWP structure 22,for both cases of the self-attraction and self-repulsion.All the extensive work on the SSB outlined above wasperformed in settings based on usual linear potentials of thedouble-well type. The objective of the present work is topropose another physical framework, in which the SSB canbe predicted in an effective nonlinear double-well potentialNDWP, induced through a spatial modulation of the localnonlinearity coefficient. Following the terminology commonly adopted in the solid-state theory 33, this nonlinear ~~ingredient of the physical model may also be a called apseudopotential~~.

In BEC settings, a pseudopotential structure may bereadily induced through spatial modulation of the local value of the s-wave scattering length, asx, which determines theeffective BEC

nonlinearity. The modulation can be implemented, through the Feshbach resonance, by means of a spatially inhomogeneous dc magnetic field 34, or by a resonant optical field, as predicted in Ref. 35and demonstratedexperimentally in Ref. 36. It was also proposed to controlthe Feshbach resonance by dint of dc electric field 37,which can be easily made inhomogeneous too. The attractiveand repulsive interactions between atoms correspond to as0 and a s0, respectively; both signs, as well as signchanging patterns, can be used to engineer effective nonlinear potentials. So designed pseudopotential lattices have attracted muchinterest in studies of BEC. In the 1D geometry, solitons,extended wave patterns, and various dynamical states supported by such structures were studied theoretically 38,39a random nonlinear lattice 40and pseudopotentials generated by a spatially monotonous ramp of the local scatteringlength 41were explored too. Recently, similar states werealso considered in nonlinear optics, assuming a periodicmodulation of the local Kerr coefficient 42. Some but much fewerresults were obtained too for 2D settings 43.However, to the best of our knowledge, SSB phenomena in nonlinear pseudopotentials have not been studied yet. Inthis work, we focus on such effects in NDWP settings, whichcan be engineered by means of techniques mentioned above,using attractive interactions between atoms in BEC, or theself-focusing nonlinearity in optics, as briefly described below. In Sec. II, we formulate the model and give estimates ofcharacteristic values of related physical parameters. Section III reports full analytical solutions



corresponding to symmetric, antisymmetric, and asymmetric states trapped by theNDWP, in the limit case when the modulation of the localnonlinearity coefficient is represented by a set of two Diracdelta functions. The relevance of the latter model is stressed,in particular, by the recently introduced 39BEC modelwith a periodic nonlinear potential of the Kronig-Penneytype, whose simplest version reduces to a periodic array ofdelta functions SSB effects were not studied in Ref. 39 .In Sec. IV, we present numerical results for the generalmodel, in which the delta functions are replaced by a pair of Gaussians of a finite width. In that case, the trapped statesare found in a numerical form, and their stability is studied by means of direct simulations of slightly perturbed stationary states and also, independently, through the computationof respective stability eigenvalues for small perturbations.The result is that asymmetric states, which are unstable in thedelta-function limit, can be readily stabilized in the generalmodel. In addition, antisymmetric states may be stabilizedalso, in two disjoint regions of the parameter state, givingrise to a bistability involving antisymmetric and asymmetricstates. A condition for the existence of symmetric states is that their norm must exceed a certain threshold value, interms of their norm, an explanation to which is given. The existence and stability of all states, including a line of theSSB bifurcation, are summarized in a single diagram, whichis presented in Sec. IV also. Results reported in this paperand perspectives for further work are summarized in Sec. V.



Model

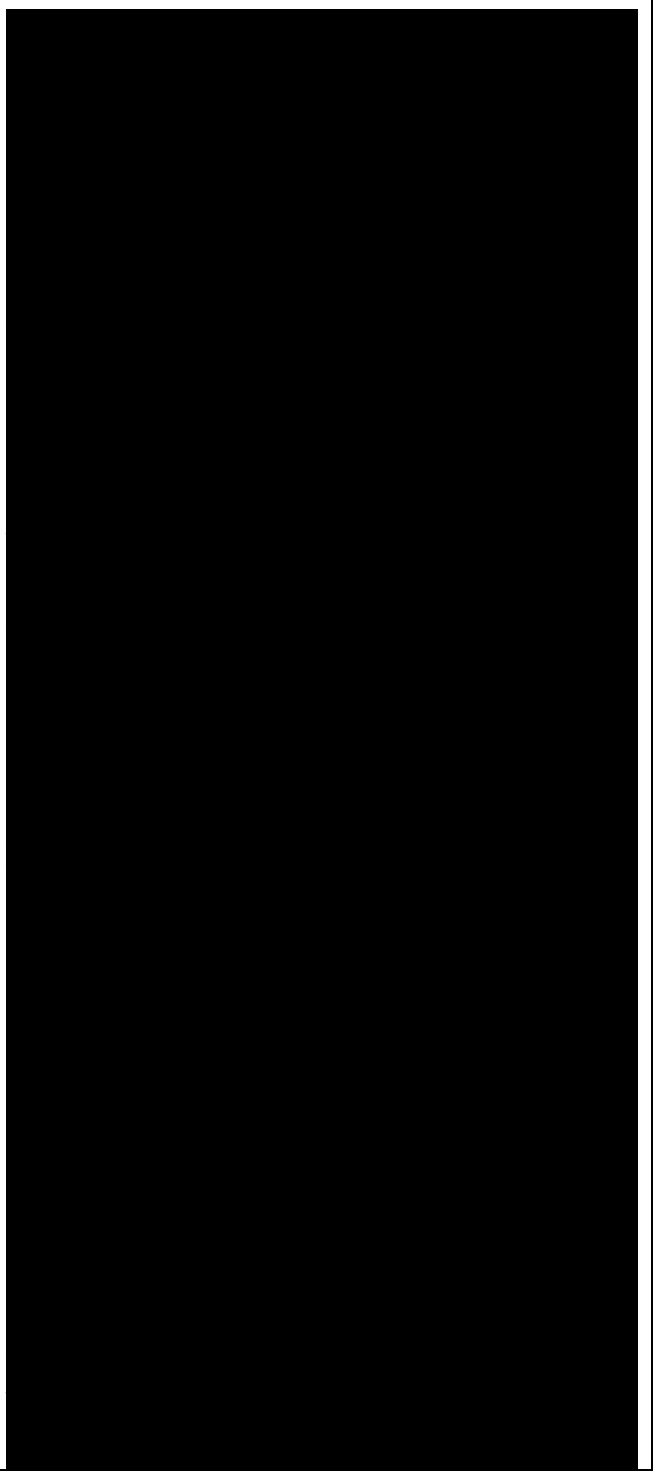
The underlying 3D Gross-Pitaevskii equation for themean-field wave function.

For physical parameters relevant to experiments with thecondensate of 7Li atoms 45, i.e., a2 m, A00.5 nm, and 20 m, characteristic values of the number of atomsin various patterns reported below see, in particular, Figs.3dand 9fall into the range of N between 200 and 1000,which is quite sufficient for experimental manipulations andobservation of the patterns. In the same range of physicalparameters, t=1 in Eq. 5is estimated as being tantamountto 10 ms, hence typical time scales for the instability development or intrinsic oscillations of breathers induced bythe instabilities, which are reported below, are expected to be in the range of 0.1–1 s, which is realistic to the currentlyavailable experimental techniques 45.

~~In terms of optical settings, a set of two narrow of widthl1 mparallel stripes with~~ strong local nonlinearity can be built, in a planar waveguide, by means of known nanotechnological methods. In that case, the power of the laser beam necessary for the self-trapping of transverse nonlinearpatterns in the waveguide made of silica may be 500 kW46, while using AlGaAs, one may reduce the necessarypower to the level of 1 kW 47. In these settings, the characteristic evolution length of the spatial beam can be madeshorter than 1 mm. Obviously, transitions between states ofdifferent types reported in this paper may be relevant to thedesign of power-controlled optical-switching schemes. Onthe other hand, the description of the planar waveguide with the pair of embedded

stripes may require a model more general than the one studied here, as it will plausibly combine the transverse modulation of the local nonlinearity with asimilar linear potential which is briefly described at the endof the next section, as the material difference between thestripes and host medium ought to affect the linear index ofrefraction also.

The analytical solution given by Eqs.13, 14, and 24make it possible to plot the bifurcation diagrams in the planes of ,and N,, which are represented by curvespertaining to a=0 in Figs. 3a–3c. To generate the diagrams, partial norms Nin expression 25for the asymmetric solutions were computed numerically analytical expressions for them are available, but they are very messy, cf. Eq.21for the symmetric and antisymmetric states. A salient peculiarity of the SSB bifurcation for a=0, evident in Fig. 3c, is its subcritical character, which means that the branches of asymmetric solutions emerge at the bifurcation point as unstable ones, and go in the backwarddirection. A subcritical bifurcation also occurs in the abovementioned model of the dual-core nonlinear fibers 10, butin that case the asymmetric branches quickly turn in the forward direction, getting stabilized at the turning point. A remarkable feature of the present model with a=0 is that this does not happen, i.e., the bifurcation in this model may becalled a “fully backward” one: the branches of the asymmetric solutions keep going backward up to the limit of =1,which corresponds to the asymmetric solutions with =−and N=1, as shown in the following subsection. Indeed,=−=1 follows from the fact that the amplitude



appertaining to the lower sign between the radicals in Eq. 24vanishes in the limit of →−.

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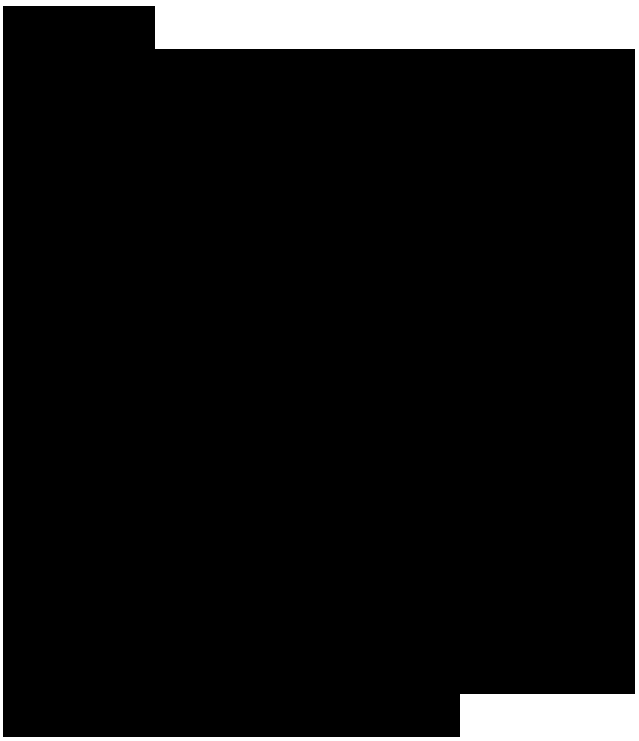
In accordance with general properties of the subcriticalSSB bifurcation 10, the symmetric solution is expected to be stable below the bifurcation point at NNbif, see Eq.23 , and unstable above it. The asymmetric branches emerging at N=Nbif are unstable as long as they go backward. In the present case a=0, this means they are alwaysunstable, as the respective branches in Figs. 3band 3cnever turn forward. All these expectations are completelyborne out by the stability analysis performed by means of both direct simulations and computation of stability eigenvalues, at finite but small values of a technical details of theprocedure are described in the next section. In particular, atNNbif the unstable symmetric state spontaneously transforms into a strongly asymmetric breather (một sóng phi tuyến có năng lượng tập trung theo kiểu cục bộ hoặc dao động) which featuresirregular oscillations, but remains robust as a whole quitesimilar to an example displayed below in Fig. 6 for a=0.7.On the other hand, unstable asymmetric states transformthemselves into breathers which maintain the original asymmetry of the unstable state, as shown in Fig. 4.Lastly, all antisymmetric states in the model with deltafunctions are unstable too. Their instability is similar to thatshown below in Fig. 7bfor a=1, transforming them intostrongly asymmetric breathers. As shown in the next section,both asymmetric and antisymmetric states may be stabilized in the general model, with finite a.

The existence of the asymmetric states, i.e., the presenceof the SSB effect in the present model, can be easily explained by the consideration of the above-mentioned limit of→−. Indeed, in this limit, the spatial scale of the solution,which is −1̃2 according to Eq. 13, is much smaller thanthe separation between the two delta functions, 22.Therefore, the full solution effectively splits into a superposition of those independently supported by each delta function in isolation. Further, it is obvious that, for given large, Eq. 12with an individual delta function gives rise totwo solutions: a trivial one.

where or note that the norm of solution 26is N=1, for any . The corresponding symmetric and antisymmetric states are built, respectively, as superpositions ofsolutions + or, equivalently, −centered at x=−1 andx= +1, or centered at x=−1 and + centered at x= +1. Asymmetric solutions are represented, in the same limit, by a superposition of solution centered at x=−1 and zero solution around x= +1, or vice versa. Of course, finding hebifurcation point requires one to perform the analysis of themodel at finite , as done in the analytical form above for thecase of the delta functions, and will be done in a numericalform below for the general case of finite a in Eq. 6.It is relevant to compare the above exact results withthose which can be easily obtained in the linear counterpartof the model, i.e., the one with the DWP based on the set oftwo delta functions; as mentioned above, such a linear potential may be a plausible ingredient of a more general model, relevant to the description of NDWP settings in optics. The stationary version of the linear equation reduces to.

B. Results

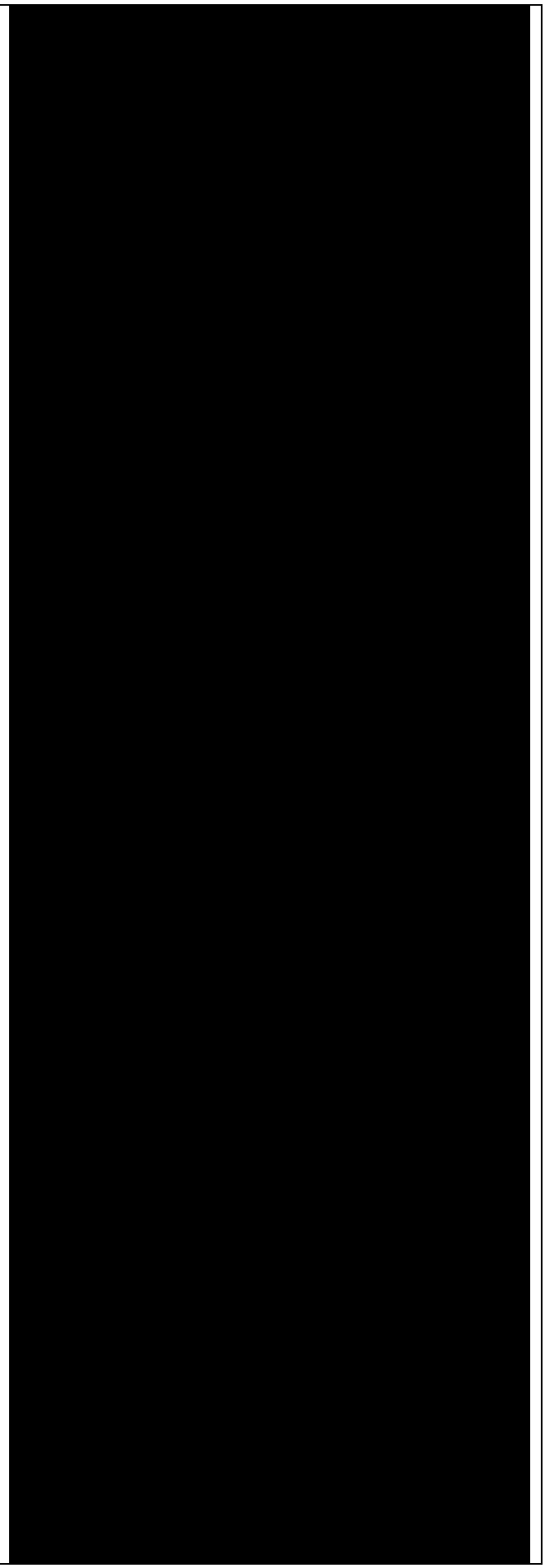
The first significant change against the results reportedabove for the model with the delta functions a=0, which happens with the increase of a, is quick stabilization ofasymmetric states with larger values of the norm, while oneswith smaller N remain unstable, originally. At a0.2, thesymmetric states are stable for all values of N at which theyexist. Another notable feature of the bifurcation diagrams atfinite a, demonstrated by Figs. 3band 3d, is that the normat which the SSB bifurcation takes place, Nbif, first decreases with the growth of a from small values up to a0.8, andthen increases with the further growth of a.



Close to their stabilization threshold in particular, ata=0.2, asymmetric states with a smaller norm, which are still unstable, demonstrate a scenario of the instability development different from what was shown by their counterpart in Fig. 4 in the case of very small a. Namely, slow regularoscillations, observed in Fig. 5 in this case, imply a dynamical resymmetrization of the unstable asymmetric state. Indeed, densities x2, taken at points x= 1, perform identical periodic oscillations, with a phase shift of betweenthem, as shown in Fig. 5b.

The stabilization of the asymmetric states at small finitevalues of a is explained by the change in the character of theSSB bifurcation: at a0, there appear turning points onbranches of asymmetric solutions in the bifurcation diagram,cf. Fig. 3c. Past the turning point, the branch

goes forwardas a stable one. In fact, Fig. 3cdemonstrates a quick transformation, with the increase of a, of the subcritical bifurcation into a supercritical one. When the bifurcation is supercritical, branches of the asymmetric solutions emerge asstable ones at the bifurcation point, and immediately go forward.We do not display the quick transition from the sub-tosupercritical bifurcation in full detail, as it actually happensat very small a, in the range of a0.1. The physical estimates given in Sec. II suggest that so small values of thescaled width of the nonlinear-potential wells correspond tophysical widths 1 m. It seems doubtful that theFeshbach-resonance technique would allow one to create astrong local inhomogeneity of the scattering length on such asmall scale nevertheless, the exact analytical solutions obtained for a=0, which provide clear clues for the understanding of the general model, are definitely relevant. An additional problem impeding the full analysis of the case of verysmall a is that, in this case, the accumulation of systematicnumerical results requires very heavy simulations, as the step size of the spatial grid must be made much smaller than a.Above the bifurcation point, symmetric states found at finite a demonstrate the familiar SSB instability, spontaneously transforming themselves into slightly nonstationary robust modes breathers, quite close in their shape to respective stable asymmetric solitons. A typical example of thistransformation is displayed in Fig. 6.As concerns antisymmetric solutions, both stable and unstable ones have been found at finite a, as illustrated by Figs.7 and 8. Figure 7bshows that the density profile of unstable antisymmetric states evolves from the double-peakpattern into



an asymmetric single-peak one, which features persistent intrinsic oscillations. This outcome of the instability development complies with the fact that the instability ofthe antisymmetric states is oscillatory, being accounted for by a quartet of eigenvalues, as seen in Fig. 8b. In otherwords, the transition from stable to unstable antisymmetric states may be considered as the Hamiltonian Hopf bifurcation 48.Figure 9 displays a combined diagram in the plane of thenorm of the solution and width of the nonlinear potentialwells, N and a, which summarizes the existence and stabilityresults for the states of all three types—symmetric, asymmetric, and antisymmetric ones. The dashed-dotted line in thefigure designates the symmetry-breaking bifurcation. Solely symmetric states exist below this line they are stable in thatregion, and stable asymmetric states exist above the line,where the symmetric ones are unstable. Solid curves in Fig.9 depict stability borders of antisymmetric solutions. For the reasons explained above, the region of very smallvalues of a, where the “quick” stabilization of asymmetricstates takes place, is not included. However, the region of theexistence of the analytical symmetric and antisymmetric solutions in the model with delta functions a=0, and the respective bifurcation point, as given by Eq. 23, are shownby the bold vertical segment and square-marked dot on theaxis of a=0 recall that the exact asymmetric solutions areunstable above the bifurcation point in the model with a=0.

~~Because the modulation profile 6does not feature thedouble-well structure for~~

~~a2 see Fig. 1, the SSB bifurcation tends to disappear as aapproaches 2.~~ Actual resultsare included in Fig. 9 for a1.35, as the convergence of thenumerical scheme becomes poor for values of a still closer to2.The chain of circles in Fig. 9 designates the thresholdminimum normNmin, necessary for the existence of symmetric states in the model. As mentioned above, in the caseof a=0 the exact threshold is Nmina=0=1̃2, and it is observed in Fig. 9 that the threshold remains in the ballpark ofthis value at finite a, which can be easily explained. Indeed,the minimum of N is attained at →−0, in which limit thespatial scale of the wave function, 1̃2, is much larger than the size of the NDWP structure, 22. Thus, from theviewpoint of this weakly localized wave function, modulation pattern 6looks like 2x. In the corresponding approximation, the wave function takes the form of expression26divided by 2, and the respective norm is, indeed, 1̃2.For NNmin, the condensate confined to the trap of largelength L i.e., in the thermodynamic limitwill tend to form a quasiuniform nearly linear state, with x=ÑL. As follows from Eqs.8and 6, the energy of the small-mplitude uniform state isH0− N2̃L4. 33In fact, this state realizes a minimum of the energy cf. Fig.10, i.e., the system's ground state. Nevertheless, a wellknown fact is that dynamically stable localized states ifferentfrom the ground state, such as the above-mentioned gapsolitons in the repulsive condensate 30, or their broadercounterparts in the form of the so-called gap waves 49, canbe created in the experiment.

~~Another notable feature observed in Fig. 9 is the bistability, i.e., the coexistence of~~

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~~stable~~ asymmetric and antisymmetric states above the stability border of the latter state. Infact, the bistability always takes place when antisymmetricstates are stable. It is interesting too that the stability area forthe antisymmetric states consists of two separate regions.Finally, it is relevant to mention that, as well as in the analytically solvable model with the delta functions $a \to 0$, theantisymmetric states never undergo a bifurcation at finite a.We stress that the stability borders displayed in Fig. 9were identified by means of direct simulations and the computation of stability eigenvalues, both methods yielding identical results. In particular, the sets of eigenvalues displayed in Fig. 8 clearly confirm the presence of two disjoint stabilityareas for antisymmetric states.

In the case of the bistability involving the asymmetric andantisymmetric states, it is interesting to compare their energies values of the Hamiltonian. To this end, Fig. 10 displays a typical example of the dependence of on norm N.The situation observed in this figure is also true in the general case: stable antisymmetric states realize smaller valuesof H than the asymmetric counterparts coexisting with them.However, as argued above, dynamically stable states can becreated in the experiment even if their energy is higher thanin some competing states. In particular, Fig. 6 demonstratesthat an unstable symmetric state definitely self-traps into an asymmetric robust breather which is close to a stable stationary solution, despite the fact that a stable antisymmetricstate exists at the same values of N=10 and a=0.7, as seenfrom Fig. 9.Note that all curves in Fig. 10 start from finite thresholdvalues of N corresponding, as said above, to the

minimumnorm Nminnecessary for the existence of the respectivestates. In particular, for the branch of symmetric solutions, Nmin is close to 12̃, as argued above cf. the existence border in the bottom of Fig. 9, while the asymmetric branch originates at the bifurcation point in agreement with thelocation of the respective dashed-dotted line in Fig. 9, atwhich the symmetric solution loses its stability. The branchof antisymmetric solutions features a fold in Fig. 10 in theregion where these solutions are unstable, which is similarto the above-mentioned fact that dependence Nin Eq.21for the unstable exact antisymmetric states has a minimum, Nantisymmin1.84 at −0.58. If replotted in termsof H and N, Eq. 21features a similar fold, at N=Nantisymmin.

V. CONCLUSION

In this work, we have introduced a model of the NDWP,alias a double-well pseudopotential, which can be created inBEC, by means of the spatially inhomogeneous Feshbachresonance, and also in nonlinear optics. The model providesfor a previoulsy unexplored setting in which effects of theSSB can be studied.

~~In the limit when each potential well is induced by thedelta function,~~ full analytical solutions were obtained for symmetric, antisymmetric, and asymmetric states. The symmetric states are stable in that case up to the symmetrybreaking bifurcation point, but beyond the bifurcation bothsymmetric and emergent asymmetric states are unstable. Inparticular, the asymmetric configurations transform themselves into

breathers. The instability of all the stationaryasymmetric states in the model with the delta functions isexplained by the fact that the respective SSB bifurcation is ofa “fully backward” type, with branches of the asymmetricsolutions never turning forward. All antisymmetric states areunstable too, in this limit form of the model.The increase of the width of the potential wells readilystabilizes the asymmetric states, which concurs with thechange of the character of the SSB bifurcation from sub- tosupercritical. Close to the stabilization border, unstable asymmetric states develop slow intrinsic oscillations, featuring effective dynamical resymmetrization.

Antisymmetricstates may also be stable in the NDWP structure with a finitewidth of the wells, which implies the bistability betweenasymmetric and antisymmetric states. The symmetric statesexist above a finite hreshold, in terms of the norm numberof atoms in the condensate, and they develop the usual SSBinstability above the bifurcation point. A simple explanationto the existence threshold was given, and an integrated diagram for the existence and stability of the trapped states ofall three types has been produced.

The analysis presented in this work suggests new experiments in the matter-wave and nonlinear-optical ettings. Theanalysis can also be developed in other directions. In particular, it may be interesting to study a two-dimensionalnonlinear-DWP configuration. In the 2D space, a triangularconfiguration with three nonlinear pseudo-potential wellsmay be considered too.
