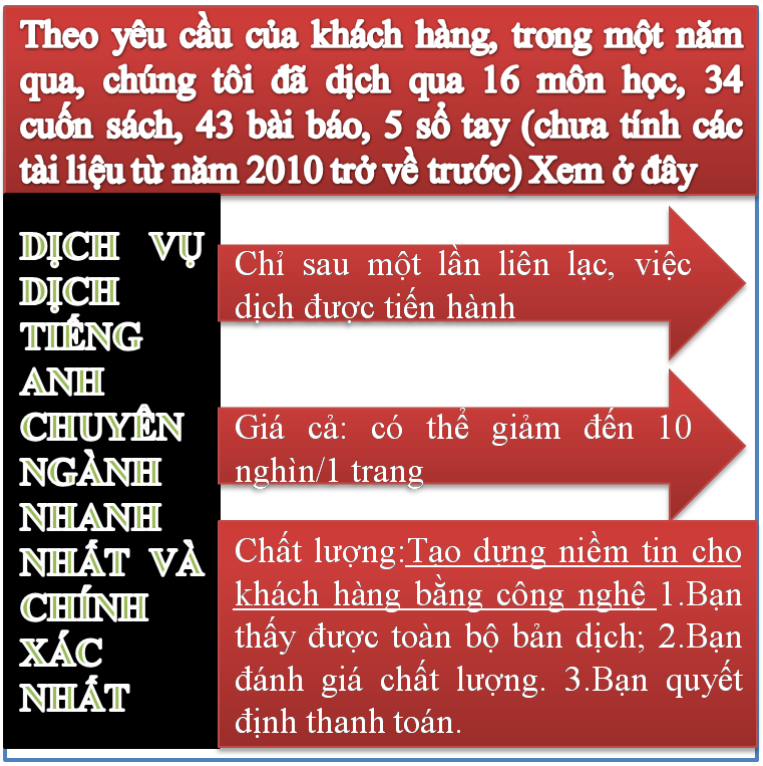

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On Partitions of a Finite Set

ABSTRACT
A pair of partitions nlt of a finite set S into disjoint non-empty subsets will be called conjugate if for each se S, the ordered pair (Vi (.?), v2 (s)) determines s, where v,(i) denotes the cardinality of the subset of n{ to which s belongs. In this note we show that S has a pair of conjugate partitions if and only if the cardinality of S is not equal to 2, 5, or 9. Partitions of this type provide a short solution to a problem arising in circuit theory.

INTRODUCTION
Suppose we have a cable consisting of n indistinguishable wires with terminals at two points A and B, and suppose for each terminal at A it is desired to identify its mate at B. We shall assume that the only operations available for making such an identification are interconnecting sets of terminals at one end and testing for current flow in the terminals at the other end. For example, if all terminals at A are connected together, then a current can flow between any two terminals at B. Without this assumption, the desired identification would present no problem, since if we denote the terminals at A by Aiy 1 < / < n (and similarly for B), then we simply test to see if a current can flow between A1 and Blt Ax and B2, ■ ■ ■, until we find a Bix such that a current can flow between Ax and Band consequently we know A{ and Bi% represent the same wire. We then use the same procedure on A2, etc. For long cables, we shall restrict ourselves further to

Về phân hoạch của một tập hợp hữu hạn

Tóm tắt
Một cặp phân hoạch π_1, π_2 của một tập hợp hữu hạn S thành các tập hợp con không rỗng không giao nhau sẽ được gọi là liên hợp nếu đối với mỗi $s \epsilon S$, cặp có thứ tự $(v_1(s), v_2(s))$ xác định s, ở đây $v_i(s)$ biểu diễn các phần tử của tập hợp con π_i chứa s (mà s thuộc nó). Trong bài giảng này, chúng tôi chứng tỏ rằng S có một cặp phân hoạch liên hợp khi và chỉ khi số phần tử của S không bằng 2, 5, hoặc 9. Phân hoạch loại này giúp chúng ta giải quyết nhanh chóng một bài toán trong lý thuyết mạch.
không giao nhau =không có chung phần tử

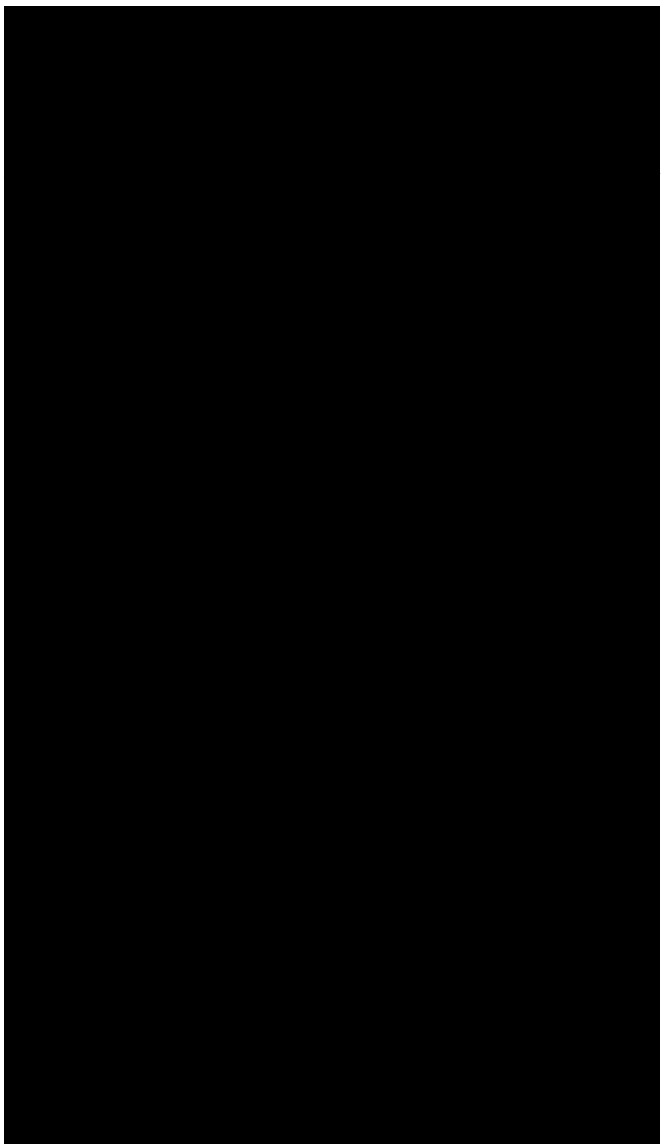
[redacted]

procedures of the following type:
Certain connections are made at A. We then go to B and make tests and, using the test results, certain connections. We finally come back to A, disconnect the connections initially made, and perform further tests. The information now in hand should be enought to determine for each j the terminal pairs Aj and B, . of wire j.
The following ingenious algorithum for solving this problem is due to my colleague K. C. Knowlton. Before presenting the general solution, a typical specific example will be given. Consider the case n = 6. Define the partitions P: {1,2,3}, {4,5}, {6}, and P': {1,4,6}, {2,5}, {3}, of the integers {1, 2, 3, 4, 5, 6}. If we let f(i) denote the number of elements in the set of P which contains i and f'(i) the similar function for the partition P', then we have the table
We note that in the table all the ordered pairs (f(i),f(i)) are distinct. To identify the terminals in our cable of six wires we first label those at A by Alt A2,..., Ae and connect them as shown in Figure 1(a). We now test at B to decide which wires have been connected at A.
FIGURE 1
Suppose, for example, we find that we have the situation indicated in Figure 1(b) where two wires with the same symbol are joined at A. We label the x's with C1; C2 and C3 (arbitrarily), the /s with C4 and C5 and the z with C6, say, as in Figure 1(c). We next connect the terminals at B together according to the partitionP', i.e., as in Figure 1(d). Finally we go back to A, disconnect the connections initially made there, and test to decide how many wires a given wire at A has been joined to at B. For example,

suppose we find that wire Ax is now connected to exactly one other wire at B. Since we know that initially At was in a set of three wires which were connected at A and now it is in a set of two wires which are connected at B and, since the pair of (/(0»/'(0) = (3,2) occurs only for i = 2, then we can conclude that A1 must correspond to C2. Similarly if it happens that A5 is now connected to two other wires at B (so that Ah belongs to a set of three wires which have been joined) then A5 must correspond to C4, etc.

The general solution may be described in the following way. Let In denote the set of integers {1,2Suppose there exists a pair of partitions of In, say P: Pu P2,. .., Pk and P': P/, P2',.. . ,P'k,, such that, if / (j) demotes the cardinality of the subsets Pt which contains j (with /'(/) defined similarly), then the map /—>■ (/(./),/' (./)) is a 1 — 1 mapping of In into In X h- We shall call such a pair of partitions conjugate. For 1 < j < k, we first connect all wires together at A which have subscripts that belong to the same Pj. We then go to B and, by suitable testing, determine the subsets S±,. . ., Sk of the Bt which have been connected together at A. We next relabel the Bt by C1; C2,. .., Cn so that for any Sr the set of indices of the Ci which occur in Sr is exactly one of the Pj. Now we connect the Ct together to form subsets ……, in such a way that, for any j, the set of indices of the C,: which are in Tj is just P-.

Finally we go back to A, disconnect all connections previously made there, and by suitable testing decide which A t have been



connected at B. We are now in a position to determine which labels represent the same wire. For if we take any wire, say Au, we know that at B it belongs to a Tj which has, say, p elements (where we can determine p). Since we also know the cardinality of the Sk to which Au belongs, say q, then, by the way the St and Tt were constructed and by the hypothesis that all the pairs (p, q) = (/(m), /' (w)) are distinct, we can determine the unique Ciu and hence the Bju such that Au and Bj represent the same wire.

It is the purpose of this note to prove that pairs of conjugate partitions exist for In if and only if n 96 2, 5, or 9. We also give a simple construction of a pair of conjugate partitions for IH for each n^2,5, or 9.

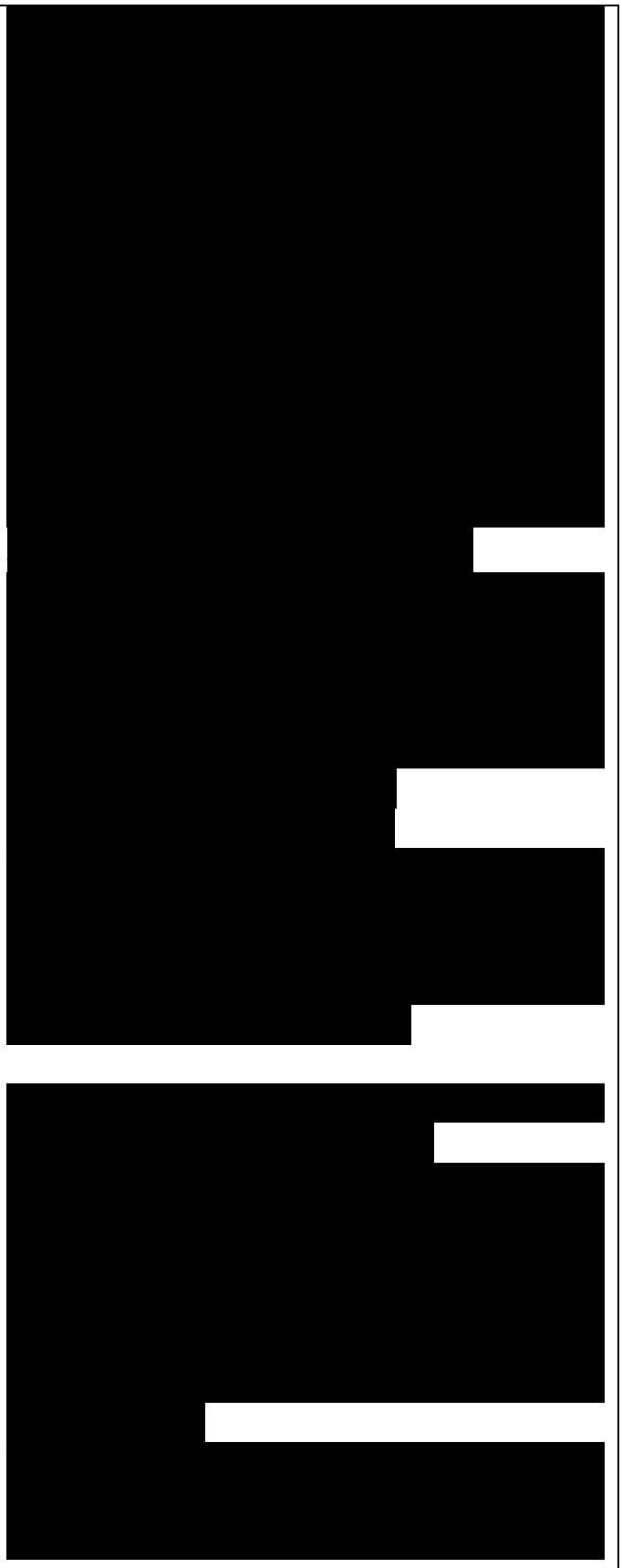
SOME NECESSARY CONDITIONS

As usual let | A | denote the cardinality of the set A. \fP: Pu P2,... ,Pk is a partition of In let C(P) denote the set {| Pi |, | P2 |,..., | Pk |} where we shall assume from now on that | jPx I < | | < - - * < | |.

LEMMA 1. IfPandP' are conjugate partitions of In then \ Pk | = | Pk, |.

PROOF. Suppose \Pk\ = m. Since P and P' are conjugate then C(P') must contain at least m distinct elements. Consequently \Pk> \ >m (since we assume that I iy I < • • • < I Pi' I). Applying the same argument to P' we see that | Pk | < | Pk, | < | Pk | and the lemma follows.

If P is a partition of In for which there exists a partition P' of /„ such that P and P' are conjugate then we shall call P admissible.



LEMMA 2. If P is admissible and \Pk\ = m then C(p) = {1,2,..., m}.
PROOF. Suppose there exists j such that 1 < j < m and j $ C(P). Since | Pk, | = m by Lemma 1 then we must have | C(P) | > m (since P is admissible). Therefore \Pk\>m+ 1, which is a contradiction.
Lemma 3. Suppose P is admissible, \Pk \ = m and n}- denotes the number of Pi such that \ Pt | = j. Then »/< [m/ j] (where [x] denotes the greatest integer not exceeding x).
PROOF. Suppose there exists j such that > [mj7] and let P' be a partition of /„ which is conjugate to P. Since r s Pa, s e Pb and | Pa |— I Pb I — J imply that f'(r) 7^ f'(s) then we must have | C(P') \ > j • n}-. But by Lemmas 1 and 2 (since P' is also admissible)
……

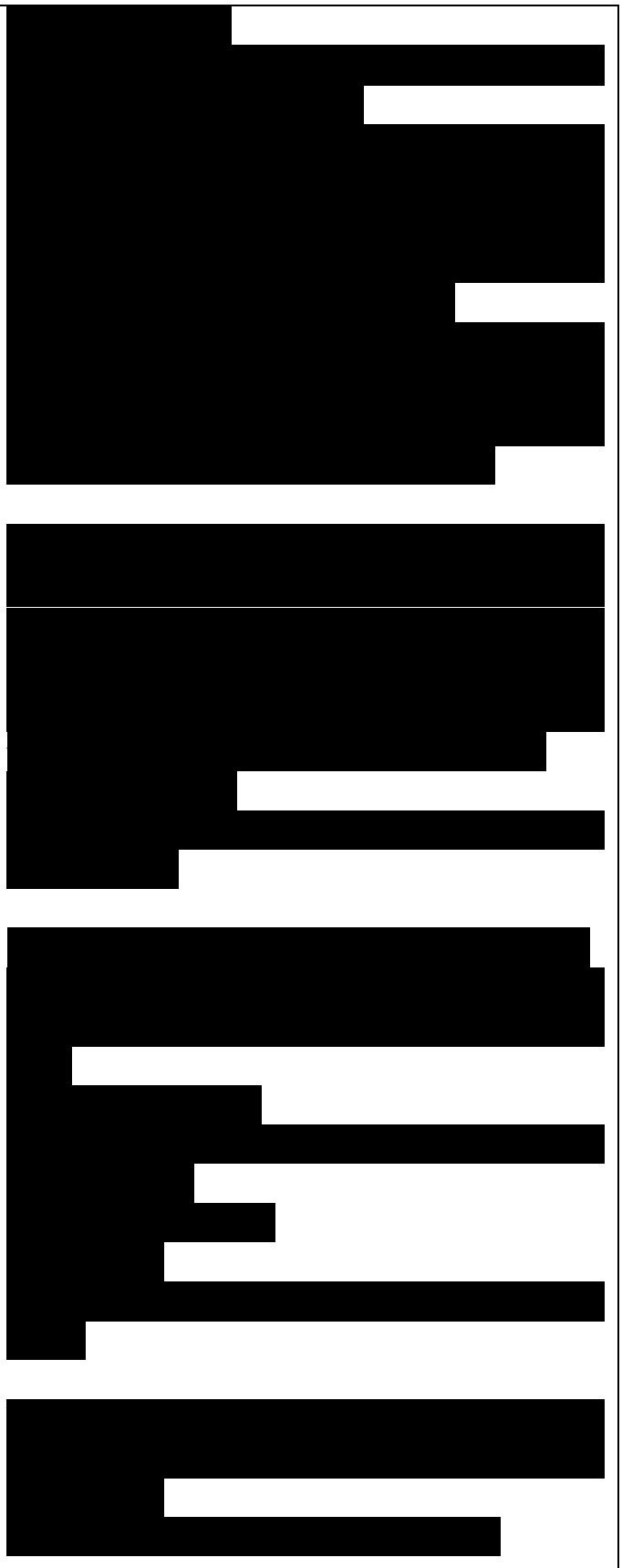
which is a contradiction. This proves the lemma.
We combine these lemmas to obtain

THEOREM 1. If P is an admissible partition of In and \ Pk \ = m then we have
……

PROOF. The left side of (1) follows from Lemma 2 since C(p)= {1,2

implies that
……..: The remaining part of (1) follows from Lemma 3 since

If we let J(m) denote JjjLi j[m/ j] and A(m) denote [m{m + l)]/2 then we have
……, and……. for m > 4.



This shows that no admissible partitions exist for h, h, or Ia.

SOME SUFFICIENT CONDITIONS

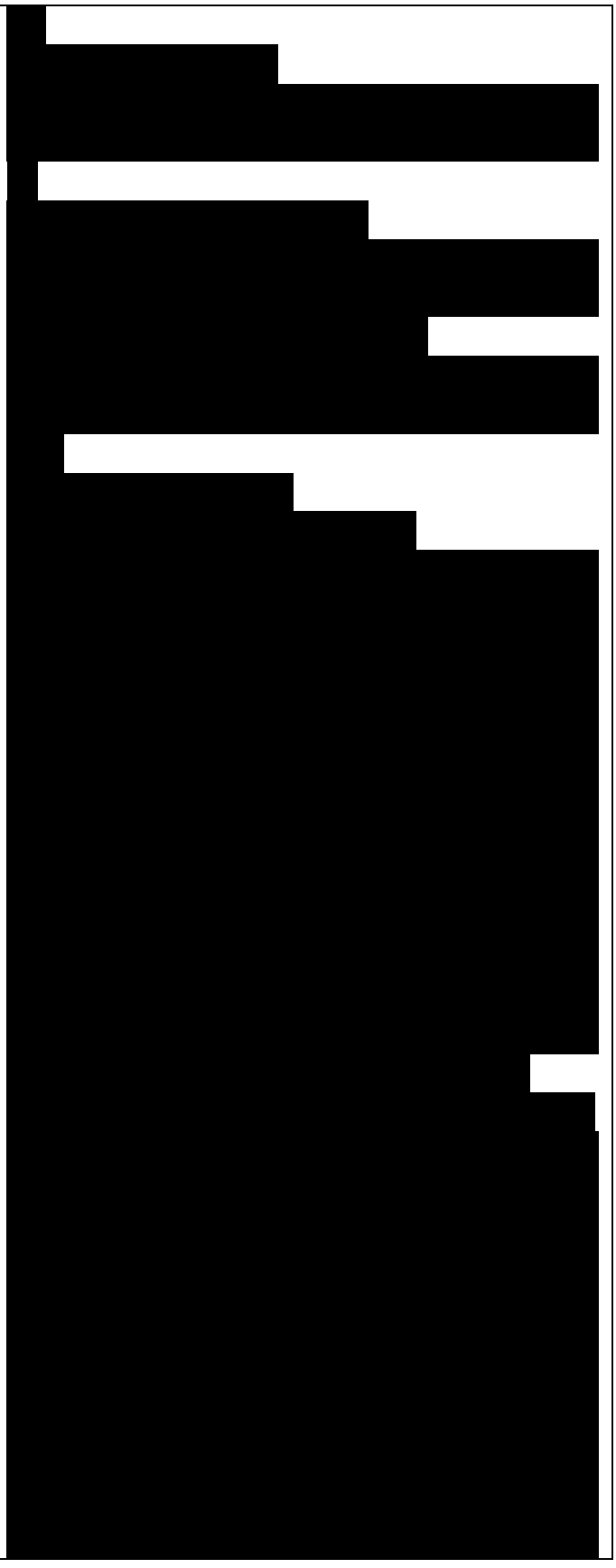
THEOREM 2. If n satisfies A(m) < n < zl(m + 1) — 2 for some positive integer m then there exist conjugate partitions of I,,.

PROOF. If we let r denote n — A(m), then 0 <r <m — 1. Consider the following array:

…….

A row (column) headed by j will be said to be "open" if there are less than j x's in that row (column). By "projecting" an x of the array, we shall mean deleting a particular x of the array, replacing it by * and placing a new x in the same row (column) and also in the first open column (row). Note that projecting an x increases the number of x's in the array by 1. The "coordinates" of an x in the array are denoted by the ordered pair (/, j) where i and j are the headings of the column and row, respectively, to which x belongs. In the diagram, the x with coordinates (2, m — 1) has been projected. The two new x's formed have coordinates (1, m — 1) and (2,1).

We now start with the original triangular array of x's where we assume that the number of rows which have a heading of 1 is r + 1 (and similarly for columns). We next project the x's which have coordinates (j, m — j -\- 1) for 2 < / < min (r + 1, m — 1). If r = m — 1 then in addition we place an x at the intersection of the last row and last column (so that this x has coordinates (1,1)). The new array now has A ini) + r x's in it and it is not difficult to see that with this



construction each x has a unique pair of coordinates. We form two partitions P and P' of In as follows: Replace the x's in the new array by the elements of In (so that every element of In is used) in an arbitrary fixed way. For each column (row) of the array, we form a subset Pj(P/) belonging to the partition P(P') by letting Pt{P-) be the set of all the integers which belong to that row (column). It follows that (f(j),f (j)) is just the pair of coordinates which j has in the array and consequently P and P' are conjugate partitions of In. This proves the theorem.

As an example of the above construction, let n = 13 = /1(4) + 3. We start with the array

…….

and project two JC'S and add the extra x (since r = 7> — m — l)to obtain

……

We arbitrarily replace the x's by the elements of /13 to form

……

from which we generate the conjugate partitions of /13:

…….

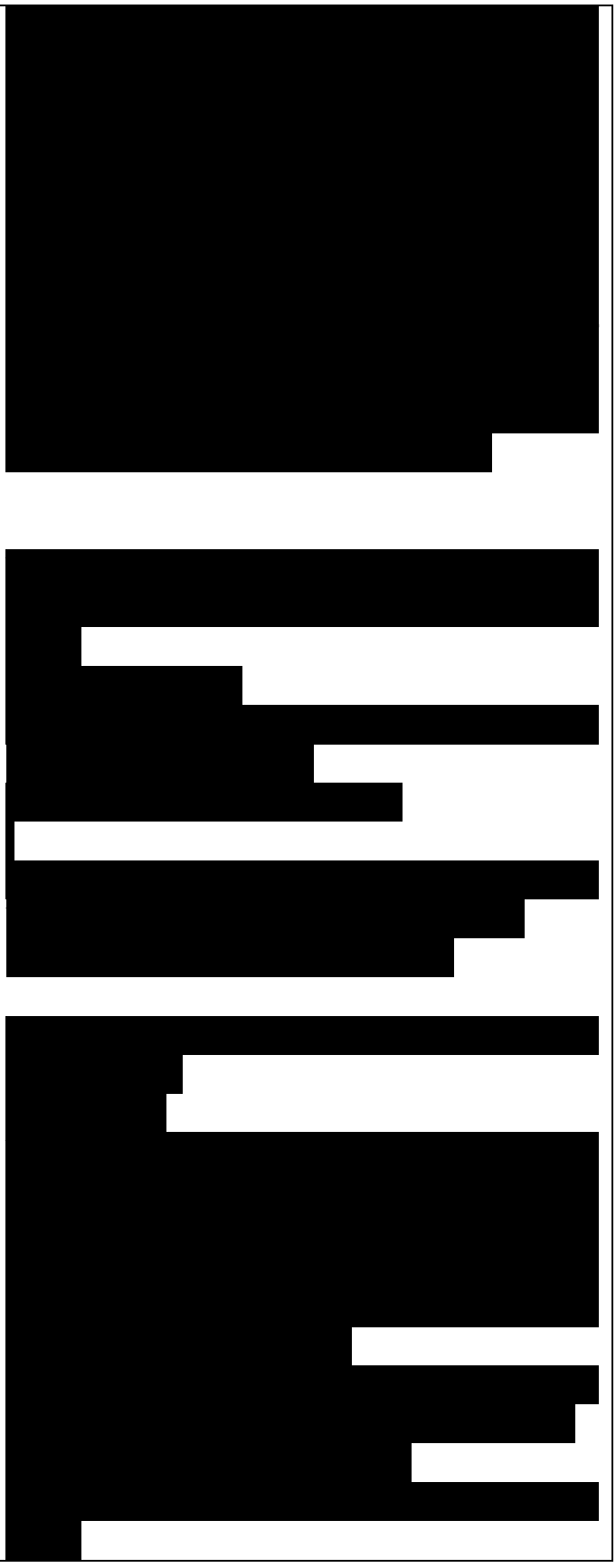
The only n for which /,, has not been shown to have conjugate partitions are those of the form A(m) — 1 and, indeed, we have already noted that no such partitions exist for I2,15, or /9. We fill this gap with

THEOREM 3. If n — A(m) — 1 for m > 4 then there exist conjugate partitions for

…….

PROOF. We start with the array

……..



and project the x's which have coordinates (j, m — j) for 2 < j < m — 2. Next we project the x at (3, m — 2) and place an additional x at the in-tersection of the last row and column (so that it has coordinates (2, 2)). Note that the new points formed from the projection of the x at (3, m — 2) have coordinates (2, m — 2) and (3, 2) which are distinct from the coordinates of any other x's in the array (since m > 4 and the x which was originally at (2, m — 2) has been projected). It is easily checked that all x's in the array have distinct coordinates so that by replacing the x's by the elements of /„ we can form conjugate partitions of In and the theorem is proved.

CONCLUDING REMARKS

It is interesting to note that, by extending the construction used in Theorem 3, it is possible to form an admissible partition P of In with | Pk | = m and n = J(m) thus achieving the upper bound derived in Theorem 1. However, it is not clear that if n is any integer such that A(m) < n < Jim) then there exists an admissible partition of In with \Pk\ — m. Since Lemmas 2 and 3 show that for any admissible partition P with] Pk | = m, we must have 1 < «,■ < [m/ j] then it might be conjectured that any partition P with 1 <nf< [m/ j] is admissible. This is not the case, however, as the following example shows. Let n = 28 and choose P so that nx = 1, n2 = 3, n3 = 2, n4 = 1, n5 = 1 and n6 = 1. Suppose P' is a partition conjugate to P and let F denote < j < 28}. We shall derive a contradiction. Since the pair (1,6) must belong to F then n2' < 2 and ns' = 1. But (6,3), (3,3), and (2,3) e F so that neither (5,3) nor (4,3)

belong to F. Hence (5,1),(5.2) , (5,4), (5,5), and (5,6) e F and therefore, since (6,4), (5,4), (3,4), and (2,4) e F then (4,4) $ F. But we must have (6,2), (5,2), (3,2), and (2.2) e F so that (4,2) $ F. Thus we have shown that (4,4), (4,3), and (4.2) cannot belong to F which is impossible. Hence P is not an admissible partition of /28.

It may be shown that, by using this procedure, the maximum number of circuit tests which must be made for the identification is essentially 2 n log2 n. On the other hand, information theory arguments show that the number of tests must be at least log2 («!)==« loga n.

It should be remarked that the function

……

can easily be put into the equivalent form

……

In this form J(m) is recognized as a well-studied number theoretic function about which statements such as

……

can be made (cf. [1]).

……

……