5.7.2 Diagnostic observer and residual generator of general form checked 7/1	5.7.2 Khối quan sát chẩn đoán và khối phát tín hiệu dư dưới dạng tổng quát
Our next task is to find out the relationships between the design parameters of the diagnostic observer and the ones given by the general residual generator	Nhiệm vụ tiếp theo của chúng ta là tìm mối liên hệ giữa các tham số thiết kế của khối quan sát chẩn đoán và các tham số thiết kế của khối phát tín hiệu dư tổng quát
(5.106)	(5,106)
whose design parameters are observer matrix L and post-filter R(p). We study two cases: s < n and s > n.	Các tham số thiết kế của nó là ma trận quan sát L và bộ hậu lọc R (p). Xét hai trường hợp: s <n và s> n.
Firstly s < n:	Trước hết s <n:
We only need to demonstrate that for s <n the diagnostic observer (5.36)- (5.37) satisfying (5.30)-(5.31), (5.38) can be equivalently written into form (5.106) . Let us define	Chúng ta chỉ cần chứng minh rằng trong trường hợp s <n khối quan sát chẩn đoán (5.36) - (5.37) thỏa mãn (5.30) - (5.31), (5.38) có thể được viết dưới dạng tương đương (5,106).Chúng ta hãy định nghĩa
(5.107) (5.108) (5.109)	(5,107) (5,108) (5,109)
and extend (5.31) and (5.38) as follows ……………………	và mở rộng (5.31) và (5.38) như sau ....
Note that choosing, for instance, T1 as a composite of the eigenvectors of A — LaC and L1 — T1 La guarantees the existence of (5.108), where La denotes some matrix that ensures the stability of matrix A — LaC. Since …………………….	Lưu ý rằng việc lựa chọn, chẳng hạn T1 là một phức hợp của các vector riêng của A - Lac và L1 - T1 La đảm bảo sự tồn tại của (5,108), trong đó La biểu diễn ma trận nào đó đảm bảo sự ổn định của ma trận A - Lac. Bởi vì ....
the residual generator ……………. can be equivalently written as …………………	khối phát tín hiệu dư .... có thể được viết dưới dạng

with …………………. We thus have the following theorem.	tương đương là .... với .... Vì thế chúng ta có định lý sau đây.
Theorem 5.14 Every diagnostic observer (5.36)-(5.37) of order s < n can be considered as a fault detection filter and post-filter V.	Định lý 5.14 Mỗi khối quan sát chẩn đoán (5.36) - (5.37) bậc s <n có thể được xem là một hỗn hợp của bộ lọc phát hiện lỗi và bộ hậu lọc V.
Remark 5.8 Theorem 5.14 implies that the performance of any diagnostic observer (5.36)-(5.37) of order s < n can be reached by an FDF together with an algebraic post-filter.	Nhận xét 5.8 Định lý 5.14 phát biểu rằng hiệu suất của bất kỳ khối quan sát chẩn đoán (5.36) - (5.37) bậc s <n nào có thể đạt được qua quá trình FDF cùng với một bộ hậu lọc đại số.
Now s > n: We first demonstrate that for s > n the diagnostic observer (5.36)-(5.37) satisfying (5.30)-(5.31), (5.38) can be equivalently written into form (5.106). To this end, we introduce following matrices ………………………..	Bây giờ s> n: Trước tiên chúng ta chứng tỏ rằng trong trường hợp s> n khối quan sát chẩn đoán (5.36) - (5.37) thỏa mãn (5.30) - (5.31), (5.38) có thể được viết dưới dạng (5,106). Để làm điều đó, chúng ta đưa vào các ma trận sau ....
and extend (5.31) and (5.38) to ………………………….. Since G is stable, there does exist T0 satisfying (5.116). Applying (5.115)-(5.118) to the diagnostic observer …………………..	và mở rộng (5.31) và (5.38) cho Bởi vì G ổn định, tồn tại T0 thỏa mãn (5.116). Áp dụng (5,115) - (5,118) cho khối

	quan sát chẩn đoán ………………….
results ………………………….	Ta được …………………………
and furthermore …………………………. (5.122)	và hơn nữa ……………………… (5,122)
which, by setting …………………….	trong đó, bằng cách đặt …………………….
finally gives …………………………..	cuối cùng ta có ………………………
We see that for s > n the diagnostic observer (5.36)-(5.37) can be equivalently written into form (5.106), in which the post-filter is a dynamic system.	Chúng ta thấy rằng khi s> n khối quan sát chẩn đoán (5.36) - (5.37) có thể được viết dưới dạng (5,106), trong đó bộ hậu lọc là một hệ thống động.
Solve equation ……………………..	Giải phương trình ……………………
for T0,T0 ,T , then we obtain …………………………………	Đối với T0, T0, T, thế thì chúng ta thu được …………………………….
The following theorem is thus proven. …………………………..	Đó cũng là phần chứng minh cho định lý sau đây. ………………………….
Theorem 5.15 Given diagnostic observer (5.36)-(5.37) of order s > n with G,L,T,V,W solving the Luenberger equations (5.30)-(5.31) and (5.38), then it can be equivalently	Định lý 5.15 Với khối quan sát chẩn đoán (5.36) - (5.37) bậc s> n có G, L, T, V, W thỏa mãn các phương trình Luenberger

written into
(5.127)(5.128)(5.129)(5.130)

We are now going to show that for a given residual generator of form (5.106) we are able to find a corresponding diagnostic observer (5.36)-(5.37). For this purpose, we denote the state space realization of R(p) with Dr + Cr(pi — Ar)~1Br. Since
……………………….

it is reasonable to define

Note that

ensure that residual generator
…………………..

satisfies Luenberger conditions (5.30)-(5.31), (5.38).

The discussion on the possible applications of the interconnections revealed in this subsection will be continued in the next subsections.

5.7.3 Applications of the interconnections and some remarks

In literature, parity relation based residual generators are often called open- loop structured, while the observer-based residual generators closed-loop structured. This view may cause some confusion, since, as known in the control theory, closed-loop and open loop structured systems have different dynamic behavior. The discussion carried out above, however, reveals that this is not the case for the parity relation and

(5.30) - (5.31) và (5.38), thế thì chúng ta có thể viết nó tương đương dưới dạng

(5,127) (5,128) (5,129) (5,130)

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

observer-based residual generators: They have the identical dynamics (under the condition that the eigenvalues are zero), also regarding to the unknown inputs and faults, as will be shown later.	[redacted]
A further result achieved by the above study indicates that the selection of a parity space vector is equivalent with the selection of the observer matrix, the feedback matrix (i.e. feedback of system output y) of an s-step deadbeat observer. In other words, all design approaches for the parity relation based residual generation can be used for designing observer-based residual generators, and vice-versa.	[redacted]
What is then the prime difference between the parity relation based and the observer-based residual generators? The answer can be found by taking a look at the implementation forms of the both types of residual generators: the implementation of the parity relation based residual generator uses a non-recursive form, while the observer-based residual generator represents a recursive form.	[redacted]
A similar fact can also be observed by the observer-based approaches. Under certain conditions the design parameters of a residual generator can be equivalently	[redacted]

converted to the ones of another type of residual generator, also the same performance can be reached by different residual generators.

[redacted]

This observation makes it clear that designing a residual generator can be carried out independent of the implementation form adopted later. We can use, for instance, parity space approach for the residual generator design, then transform the parameters achieved to the parameters needed for the construction of a diagnostic observer and finally realize the diagnostic observer for the on-line implementation. The decision for a certain type of design form and implementation form should be made on account of

[redacted]

[redacted]

- the requirements on the on-line implementation,

[redacted]

- which approach can be readily used to design a residual generator that fulfills the performance requirements on the FDI system,

[redacted]

- and of course, in many practical cases, the available design tools and designer’s knowledge of design approaches.

[redacted]

Recall that parity space based system design is characterized by its simple mathematical handling. It only deals with matrix- and vector-valued operations. This fact attracts

[redacted]

attention from industry for the application of parity space based methods. Moreover, the one-to-one mapping between the parity space approach and the observer-based approach described in Theorems 5.12 and 5.13 allows an observer-based residual generator construction for a given a parity vector. Based on this result, a strategy called parity space design, observer-based implementation has been developed, which makes use of the computational advantage of parity space approaches for the system design (selection of a parity vector or matrix) and then realizes the solution in the observer form to ensure a numerically stable and less consuming on-line computation. This strategy has been for instance successfully used in the sensor fault detection in vehicles and highly evaluated by engineers in industry. It is worth mentioning that the strategy of parity space design, observer-based implementation can also be applied to continuous time systems.	[redacted]
Table 5.1 summarizes some of important properties of the residual generators described in this section, which may be useful for the decision on the selection of design and implementation forms. In this table,	[redacted] [redacted]

• "solution form" implies the required knowledge and methods for solving the related design problems. LTI stands for the needed knowledge of linear system theory, while algebra means for the solution only algebraic computation, in most cases solution of linear equations, is needed.	[redacted]
• "dynamics" is referred to the dynamics of LTI residual generator (5.24). OEE + v implies a composite of output estimation error and an algebraic post-filter, OEE + R(p) a composite of output estimation error and a dynamic post-filter.	[redacted]
Table 5.1 Comparison of different residual generation schemes	[redacted]
5.7.4 Examples Example 5.8 We now extend the results achieved in Example 5.6 to the construction of an observer-based residual generator. Suppose that (5.97) is a discrete time system. It follows from Theorem 5.12 and (5.103) that ……………………………….	[redacted] [redacted] [redacted]
builds a residual generator. If we are interesting in achieving a residual residual generator whose dynamics is governed by ………………..	[redacted]
then the observer gain matrix L should be extended to ………………………..	[redacted]

	[redacted]
Note that in this case the above achieved results can also be used for continuous time systems.	[redacted]
In summary, we have some interesting conclusions:	[redacted]
• given a transfer function, we are able to design a parity space based residual generator without any involved computation and knowledge of state space realization	[redacted]
• the designed residual generator can be extended to the observer-based one. Once again, no involved computation is needed for this purpose	[redacted]
• the observer-based form can be applied both for discrete and continuous time systems.	[redacted]
We would like to mention that the above achieved results can also be extended to MIMO systems.	[redacted]
Example 5.9 We now apply the above result to the residual generator design for our benchmark DC motor DR300 given in Subsection 3.7.1. It follows from (3.50) that ………………………………..	[redacted]
	[redacted]
which yields	[redacted]
…………….. (5.135)	[redacted]

Now, we design an observer-based residual generator of the form ……………………………….. (5.136)	[redacted]
without the knowledge of the state space representation of the system. To this end, using Theorem 5.12 and (5.103) with vs given in (5.135) results in ……………………………..	[redacted]
To ensure a good dynamic behavior, the eigenvalues of matrix G are set to be	[redacted]
— 10.—10.—10, which leads to …………………………..	[redacted]
and further	[redacted]
5.8 Notes and references	[redacted]
The general form and parameterization of all LTI stable residual generators were first derived by Ding and Frank [38]. The FDF scheme was proposed by Beard [11] and Jones [86]. These works are recognized as marking the beginning of the model-based FDI theory and technique. Both FDF and DO techniques have been developed on the basis of linear observer theory, to which O'Reilly's book [111] gives an excellent introduction.	[redacted]
Only few references concerned characterization of DO and parity space approaches can be found in the literature. For this reason, an extensive and systematic	[redacted]

study on this topic has been included in this chapter. The most significant results are	[redacted]
• the necessary and sufficient condition for solving Luenberger equations	[redacted]
(5.30) -(5.31), (5.38) and its expression in terms of the solution of parity equation (5.87)	[redacted]
• the one-to-one mapping between the parity space and the solutions of the Luenberger equations	[redacted]
• the minimum order of diagnostic observers and parity vectors and	[redacted]
• the characterization of the solutions of the Luenberger equations and the parity space.	[redacted]
Some of these results are achieved based on the works by Ding et al. [28] (on DO) and [44] (on the parity space approach). They will also be used in the forthcoming chapters.	[redacted]
The original versions of numerical approaches proposed by Ge and Fang as well as Ding at al. have been published in [61] and [28], respectively.	[redacted]
Accompanied with the establishment of the framework of the model-based fault detection approaches, comparison among different model-based residual generation schemes has increasingly received attention. Most of studies have been devoted to the interconnections between FDF, DO on the one side and parity space approaches on the other side, see for instance, the significant	[redacted]

work by Wuenneberg [148]. Only a few of them have been dedicated to the comparison between DO and factorization or frequency approach. A part of the results described in the last section of this chapter was achieved by Ding and his co-worker [42].	[redacted]
An interesting application of the comparison study is the strategy of parity space design, observer-based implementation, which can be applied both for discrete and continuous time systems and allows an easy design of observer- based residual generators. In [131], an application of this strategy in practice has been reported. It is worth emphasizing that this strategy also enables an observer-based residual generator design based on the system transfer function, instead of the state space representation, as demonstrated in Example 5.9.	[redacted]
Perfect unknown input decoupling	[redacted]
In this chapter, we address the problems of generating residual signals which are decoupled from the disturbances (unknown inputs). That means the generated residual signals will only be influenced by the faults. In this sense, such a residual generator also acts as a fault indicator. It is often called unknown input residual generator. Fig.6.1 sketches the objective of this chapter schematically.	[redacted]

	[redacted]
Fig. 6.1 Schematical description of unknown input decoupled residual generation	[redacted]
6.1 Problem formulation Consider system model (3.29) and its minimal state space realization (3.30)-(3.31). It is straightforward that applying a residual generator of the general form (5.24) to (3.29) yields ………………….. (6.1)	[redacted]
Remember that for the state space realization (3.30)-(3.31), residual generator (5.24) can be realized as a composition of a state observer and a post-filter, …………….	[redacted]
It turns out, by setting e — x — x, ………………………..	[redacted]
which can be rewritten into, by noting Lemma 3.1, ……………………………	[redacted]
with an LCF of Cyf (p) — My1(p)Nf (p) and Cyd(p) — M^1 (p)Nd(p). It is interesting to notice that ………………………….	[redacted]
Hence, we assume in our subsequent study, without loss of generality, that ………………………..	[redacted]

	[redacted]
For the fault detection purpose, an ideal residual generation would be a residual signal that only depends on the faults and is simultaneously independent of disturbances. It follows from (6.1) that this is the case for all possible disturbances and faults if and only if	[redacted]
…………………………(6.3)	[redacted]
Finding a residual generator which satisfies condition (6.3) is one of the mostly studied topics in the FDI area and is known as, among a number of expressions, perfect unknown input decoupling.	[redacted]
Definition 6.1 Given system (3.29). Residual generator (5.24) is called perfectly decoupled from the unknown input d if condition (6.3) is satisfied. The design of such a residual generator is called .	[redacted]
In the following of this chapter, we shall approach PUIDP. Our main tasks consist in	[redacted]
• the study on the solvability of (6.3),	[redacted]
• presentation of a frequency domain approach to PUIDP	[redacted]
• design of unknown input fault detection filter (UIFDF)	[redacted]
• design of unknown input diagnostic observer (UIDO) and • design of unknown input parity	[redacted]

relation based residual generator.	[redacted]
6.2 Existence conditions of PUIDP	[redacted]
In this section, we study	[redacted]
• under which conditions (6.3) is solvable and • how to check those existence conditions.	[redacted]
	[redacted]
6.2.1 A general existence condition We begin with a reformulation of (6.3) as …………………….. (6.4)	[redacted]
	[redacted]
with A = 0 as some transfer matrix. Since	[redacted]
………………………..	[redacted]
and R(p) is arbitrarily selectable in the following theorem is obvious.	[redacted]
Theorem 6.1 Given system (3.29), then there exists a residual generator ………………………….	[redacted]
	[redacted]
such that (6.3) holds if and only if	[redacted]
……………………. (6.5)	[redacted]
Proof. If (6.5) holds, then there exists a R(p) such that	[redacted]
R(p)Mu(p)Gyd(p) = 0 and R(p)Mu(p)Gyf (p) = 0 This proves the sufficiency. Suppose that (6.5) does not hold, i.e. rank [Gyf (p) Gyd(p)] = rank (Gyd(p)) .	[redacted]

As a result, for all possible R(p)Mu(p) one can always find a transfer matrix T(p) such that R(P)jMu(p)Gyf (p) = R(p)Mu(p)Gyd(p)T (p) Thus, R(p)Mu(p)Gyd(p) = 0 would lead to R(P)jMu(p)Gyf (p) = 0 i.e. (6.3) can never be satisfied. This proves that condition (6.5) is necessary for (6.3)	[redacted] [redacted] [redacted] [redacted] [redacted] [redacted] [redacted]

4.2 Excitations and sufficiently excited systems checked 8/1 In this section, we briefly address the issues with excitation signals, which are, as shown above, needed for detecting multiplicative faults. Let G%. (p) be the fault transfer matrix of a multiplicative fault and satisfy …………………….. then we can find a K-dimensional subspace WexCj^. so that for all u £ WexCj^. % (p)u(p) = 0. From the viewpoint of fault detection, subspace WexCj^. contains all possible input signals that can be used to excite a detection procedure. Definition 4.3 Let G%. (p) be the fault transfer matrix of multiplicative fault ………………………. (4.16)	[redacted]
is called excitation subspace with respect to Mathematically, we can express the fact that detecting an additive fault, say is independent of exciting signals by defining …………………… In this way, we generally say that Definition 4.4 System (4.1)-(4.2) is sufficiently excited regarding to a fault C, if ………………… (4.17)	[redacted]
With this definition, we can reformulate the definition of the fault detectability more precisely.	[redacted]
Definition 4.5 Given system (4.1)-(4.2). A fault is said structurally detectable if for u	[redacted]

£ UeXc,£.
……………………..	(4.18)

Remark 4.1 In this book, the rank of a transfer matrix is understood as the so-called normal rank if no additional specification is given.

4.3 Structural fault isolability

4.3.1 Concept of structural fault isolability

For the sake of simplicity, we first study a simplified form of fault isolability problem, namely distinguishing the influences of two faults. An extension to the isolation of multiple faults will then be done in a straightforward manner.

Consider system model (4.1)-(4.2) and suppose that the faults under consideration are detectable. We say any two faults, ,i — j, are isolable if the changes in the system output caused by these two faults are distinguishable. This fact can also be equivalently expressed as: any simultaneous occurrence of these two faults would lead to a change in the system output. Mathematically, we give the following definition.

Definition 4.6 Given system (4.1)-(4.2). Any two detectable faults, £ — [fi Zj] ,i — j, are isolable, when for u £ Uexc£i H Uexc£.
…………………….	(4.19)

It is worth mentioning that detecting a fault in a disturbed system requires distinguishing the fault from the disturbances. This standard fault detection problem can also be similarly formulated as an isolation problem for two faults.

In a general case, we say that a group of faults are isolable if any simultaneous

[redacted]

occurrence of these faults would lead to a change in the system output. Define a fault vector

…………………… (4.20)

which includes I structurally detectable faults to be isolated.

Definition 4.7 Given system (4.1)-(4.2). The faults in fault vector £ are isolable, when for all u £ H ^exc,^i

………………. (4.21)

We would like to call reader's attention on the similarity between the isolability of additive faults and the so-called input observability which is widely used for the purpose of input reconstruction. Consider system

It is called input observable, when y(t) = 0 implies f (t) = 0. Except the assumption on initial condition x (0), the physical meanings of the isolability of additive faults and input observability are equivalent.

With the aid of the concept of fault transfer matrices, we now derive existence conditions for the structural fault isolability.

Theorem 4.2 Given system (4.1)-(4.2), then any two faults with fault transfer matrices G%. (p),G^j (p),i = j, are structurally isolable if and only if

rank [G^. (p) G^j (p)] = rank (G^. (p)) + rank (^G^j (p)^ . (4.22)

Proof. It follows from (4.11)-(4.15) that the changes in the output caused by ^,£j can be respectively written as

………………………….

where

Zi(p) = C(dfi) for & = fi or zt(p) = L (d^u(t)) for & e {9At ,0Bt ,6Ci ,0Di} with

[redacted]

u £ Uexc£i H Uexc£.. Since

………………..

it holds that if £ is not isolable, then

………………………………

The theorem is thus proven. □

An extension of the above theorem to a more general case with a fault vector £ = [£1 •••] is straightforward and hence its proof is omitted.

Corollary 4.1 Given system (4.1)-(4.2), then £ with fault transfer matrix Gz(P)= [%(p) ... G*,(p)]

is structurally isolable if and only if

In order to get a deeper insight into the results given in Theorem 4.2 and Corollary 4.1, we study some special cases often met in practice.

Suppose that the faults in fault vector £ — [^1 ■■■] are additive faults. Then the following result is evident.

Corollary 4.2 Given system (4.1)-(4.2) and assume that — 1, ■■■ ,1 < kf are additive faults. Then, these I faults are isolable if and only if

……………………..

This result reveals that, to isolate I different faults, we need at least an l-dimensional subspace in the measurement space spanned by the fault transfer matrix. Considering that rank (G%(p)) < min {to, 1} with m as the number of the sensors, we have the following claim which is very easy to check and thus useful for the practical application.

Claim. The additive faults are isolable only if the number of the faults is not

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

larger than the number of the sensors.	[redacted]
Denote the minimal state space realization of G^ (p) by ………………………………………..	[redacted]
Check condition (4.24) can be equivalently expressed in terms of the matrices of the state space description.	[redacted]
Corollary 4.3 Given system (4.1)-(4.2) and assume that ^i)i — 1, kf, are additive faults. Then these I faults are isolable if and only if	[redacted]
Proof. The proof becomes evident by noting that Recall that for additive faults the fault isolability introduced in Definition 4.7 is identical with the concept of input observability known and intensively studied in the literature, we would like to extend our study • to find out alternative conditions for checking conditions (4.24) or (4.25) • to compare them with the results known in the literature and • to gain a deeper insight into the isolability of additive faults, which will be helpful for some subsequent studies in the latter chapters.	[redacted]
To simplify our study, we first consider (p) = C (pi — A)~x . It follows from Cayley-Hamilton Theorem that (4.26) which can be rewritten into …………………………… It is obvious that if	[redacted]

……………………… then there exists a u which yields …………………………… Thus,	[redacted]

11 Integration of norm based and statistical methods 18/1 checked	11 Tích hợp các phương pháp dựa trên chuẩn và thống kê
The achieved results evidently reveal that, both in the norm based methods and the approach presented in this section, the boundedness of and the covariance of the residual signal given in (11.8) play an important role in threshold determination, as we can see from (11.18). This is a convincing argument for a system designer to make use of the degree of the design freedom offered by the observer to achieve an optimal trade-of between (dich tieng viet)	Rõ ràng, các kết quả thu được cho thấy rằng cả trong phương pháp dựa trên chuẩn và phương pháp được trình bày trong phần này, cận của δ_{r_d} và hiệp phương sai của tín hiệu dư trong (11.8) đóng vai trò quan trọng trong việc xác định ngưỡng, chúng ta có thể thấy điều này từ (11.18). Đây là một lập luận có sức thuyết phục để nhà thiết kế hệ thống sử dụng mức độ tự do thiết kế của bộ quan sát để đạt được sự dung hòa tối ưu giữa (Dich tieng viet)
Example 11.1 We continue our study in Example 10.1, where a fault detection system is designed for the three tank system benchmark. Now, in addition to the noises, off set in the sensors is taken into account and modelled as unknown inputs by (dich tieng viet)	**Ví dụ 11.1** Chúng ta tiếp tục xét Ví dụ 10.1, trong đó hệ thống phát hiện lỗi được thiết kế cho tiêu chuẩn hệ ba bể. Bây giờ, cùng với nhiễu, off set trong các cảm biến được tính đến và được mô hình hóa dưới dạng các đầu vào bất định (Dich tieng viet)
It is assumed that is bounded by d = 0.05 . Our design objective is to determine the threshold Jth using Algorithm 11.3. For the	[redacted]

residual generation purpose, we use the same two Kalman filters designed in Example 10.1, i.e. (a) a Kalman filter driven by the level sensor of tank 1 (b) a Kalman filter driven by all three sensors. Under the same assumptions with α= 0 05, we have	[redacted]
Case (a) with one sensor: Jth = 26. 3349	[redacted]
Case (b) with three sensors: Jth = 68 .0159 .	[redacted]
Fig.11.2 and Fig.11.3 show the simulation results of the testing statistic and	[redacted]
Fig. 11.2 Testing statistic and the threshold: one sensor case Fig. 11.3 Testing statistic and the threshold: three sensors case (dich tieng viet)	[redacted]
threshold by an offset fault (5cm) in sensor 1 at = 12 sec,with respect to the designed FD systems (dich tieng viet)	[redacted]
11.2 Residual evaluation scheme for	[redacted]

stochastically uncertain systems In Section 8.5, we have studied the residual generation problems for stochastically uncertain systems. The objective of this section is to address the residual evaluation problems, as sketched in Fig.11.4. 11.2.1 Problem formulation As studied in Section 8.5, we consider system model . A ,ΔB ,ΔC,ΔD, ΔE and ΔF represent model uncertainties satisfying. (dich tieng viet) FDI in systems with deterministic disturbances and stochastic uncertainties. with known matrices A_i , B_i , C_i ,D_i, E_i ,F_i ,$i=1, \ldots l$, of appropriate dimension. $p^T(k)$ =[$p_1(k)$ … $p_l(k)$] represents model uncertainties and is expressed as a stochastic process with (dich tieng viet) where ,i = 1, … l, are known. It is further assumed that p(0) ,p(1)…,are independent and (0) ,u(k) , d(k) , f(k) are independent of p(k) . For the purpose of residual generation, an	[redacted]

FDF is used. The dynamics of the above residual generator is governed by (dich tieng viet)	[redacted]
The matrices in (11.24) , (11.25)are described in Section 8.5. We assume that the system is mean square stable. In the remainder of this section, the standard variance of r(k) is denoted by (dich tieng viet)	[redacted]
It is the objective of our study in this section that a residual evaluation strategy will be developed and integrated into a procedure of designing an observer-based FDI system. This residual evaluation strategy should take into account a prior knowledge of the model uncertainties and combine the statistic testing and norm based residual evaluation schemes. Note that the residual signal considered in the last section is assumed to be a normal distributed (dich tieng viet)	[redacted]
Differently, we have no knowledge of the distribution of the residual signal addressed in	[redacted]

this section. The problems to be addressed in the next subsections are • selection of a residual evaluation function and • threshold determination for the given residual evaluation function and an allowable false alarm rate α. (dich tieng viet) 11.2.2 Solution and design algorithms A simplest way to evaluate the residual signal is to compute its size at each time instant and compare it with a threshold. Considering that is a stochastic process whose distribution is unknown, it is reasonable to set the threshold equal to (dich tieng viet) where β(>1) is some constant used to reduce the false alarm rate. In (11.28), the first term represents the bound on the mean value of the residual signal in the fault-free case, while the second term, considering the stochastic character of , is used to express the expected derivation of from its mean value.	[redacted]

(dich tieng viet)	[redacted]
It is evident that the above decision logic with threshold (11.28) may result in a high false alarm rate if the standard variance of is large. For this reason, we propose the following residual evaluation function	[redacted]
(dich tieng viet)	[redacted]
is the average of the residual signal over the time interval (k – N ,k), which is influenced by both the additive and multiplicative faults. The following theo- rem reveals an important statistical property of evaluation function (11.31).	[redacted]
(dich tieng viet)	[redacted]
Theorem 11.1 Given system model (11.24)-(11.25) and suppose that the system is mean square stable, and with are bounded. Then,	[redacted]
(dich tieng viet)	[redacted]
11.2 Residual evaluation scheme for	[redacted]

stochastically uncertain systems with.	[redacted]
(dich tieng viet)	[redacted]
and moreover, considering that the size of all eigenvalues of is smaller than one, we also have	[redacted]
(dich tieng viet)	[redacted]
where, due to the boundness of E and E , μ is a constant and independent of N. It results in finally . The theorem has thus been proven Note that	[redacted]
(dich tieng viet)	[redacted]
11.2 Residual evaluation scheme for stochastically uncertain systems	[redacted]
i.e. J will deliver a good estimate for the mean value of the residual signal. Motivated and guided by the above discussion, we propose, corresponding to evaluation function (11.31), the following general form for setting the threshold:	[redacted]
(dich tieng viet)	[redacted]
where β is a constant for a given N . In this	[redacted]

way, the problem of determin- ing the threshold is reduced to find β. Next, we approach this problem for a given allowable false alarm rate α . To this end, we first introduce the well-known Tchebyche Inequality, which says: for a given random number x and a constant A>0 satisfying . it holds	[redacted]
Recall that the false alarm rate is defined by (dich tieng viet)	[redacted]
From (11.38) it can be seen that a lower allowable false alarm rate requires a larger β	[redacted]
To complete our design procedure, it remains to find A and B as well as C and C which are needed for the computation of threshold (11.36) as well as μ in (11.32). Using the LMI technique introduced in Chapter 9, we obtain the following results.	Để hoàn thành quy trình thiết kế của chúng ta, chúng ta vẫn còn cần phải tìm A và B cũng như C và C đó là những đại lượng cần thiết để tính ngưỡng (11.36) cũng như μ trong(11.32). Sử dụng kỹ thuật LMI được trình bày trong chương 9, chúng ta thu được kết quả như sau.

On the other side, it holds that checcked 16 As a result, The theorem is thus proven.	Mặt khác, ta có: Do đó, Đó là điều phải chứng minh.
From the FDI viewpoint, the result in Theorem 12.12 can be interpreted as the fact that the FD system designed by the trade-off strategy developed in this paper is less robust in comparison with the FD system designed by using the unified solution. On the other side, as mentioned in the former subsection, the new trade-off strategy delivers a better estimation of the size of the possible faults. In this context, we would like to emphasize that the decision for a certain optimization approach should be made based on the design objective not on the mathematical optimization performance index.	Từ quan điểm FDI, kết quả trong Định Lý 12.12 có thể được hiểu là hệ FD được thiết kế bằng phương pháp thỏa hiệp trong bài báo này không bền vững bằng hệ FD được thiết kế bằng phương pháp nghiệm duy nhất. Bên cạnh đó, như đã đề cập trong phần trước, phương pháp thỏa hiệp mới có khả năng ước tính kích thước của các lỗi khả dĩ tốt hơn. Trong bối cảnh này, chúng tôi muốn nhấn mạnh rằng quyết định chọn lựa một phương pháp tiếp cận nhất định phải được thực hiện dựa trên mục tiêu thiết kế chứ không phải dựa trên chỉ số hiệu suất tối ưu toán học.
12.3.6 An example In this subsection, an example is given to illustrate the results achieved in the last two sections. Consider the FD problem of a system in the form of (12.61)-(12.62) with matrices From Theorem 12.8, we get the optimal gain matrix L1,V1 The unified solution that solves (12.78), (12.82) and (12.76) simultaneously is The optimal performance indexes, as obtained by solving (12.78), (12.82) and (12.76) are shown in Fig.12.8. It can be seen that, fig 12.8 Performance index J∞(L1,V1)=J0(L1,V1)=J1,ω(L1,V1)=J2,ω(L1	[redacted] [redacted] [redacted] [redacted] [redacted] [redacted]

,V1)≡0.1769 (dashed line), performance index $J_{1,\omega}(L_2,V_2)$ (solid line), and performance index $J_{2,\omega}(L_2,V_2)$ (dotted line)	[redacted]
These results verify Theorems 12.10-12.12.	[redacted]
In the simulation study, the simulation time is set to be 2000 seconds and the control input is a step signal (step time at 0) of amplitude 5. The unknown disturbances are, respectively, a continuous signal taking value randomly from a uniform distribution between [-0.1, 0.1], a sine wave 0.1 sin(0.1t), and a chirp signal with amplitude 0.1 and frequency varying linearly from 0.02 Hz to 0.06 Hz. Fault 1 appears at the 1200-th second as a step function of amplitude 0.75. Fault 2 appears at the 1000-th second as a step function of amplitude 0.4. The fault energy is $\lVert f \rVert_2$= 24 71. The residual signals are shown in Fig.12.9, where r_1 denotes the residual vector generated with L_1,V_1 and r_2 that by L_2,V_2.AS $\lVert r_1 \rVert$=25.45, $\lVert r_2 \rVert$=2146, theresidual vector obtained by L_1,V_1 gives a better estimation of the energy level of the fault signal. On the other side, we see from the second figure that the residual vector got by L_2,V_2 shows 12 Integrated design of fault detection systems a better fault/disturbance ratio in the sense of (12.78), (12.82) and (12.76). This demonstrates the results in Theorem 12.12.	[redacted]
Fig. 12.9 Residual signals	[redacted]

12.4 On the application to stochastic systems	[redacted]
In the last two sections, two trade-off strategies and the associated design methods have been developed in the norm based evaluation framework. It is of practical interests to know if they are still valid for stochastic systems and in the statistic testing framework. In this section, we shall briefly discuss the related problems.	[redacted]
12.4.1 Application to maximizing FDR by a given FAR In Subsection 11.1.3, we have introduced a GLR solution to the residual evaluation and threshold computation for stochastic systems modelled by (11.1)- (11.2). The core of this approach is the computation of the FAR in the sense of Definition 12.1, which is given by (see (11.17))	[redacted]
Equation (12.85) can be equivalently written as	[redacted]
when the residual evaluation function is re-defined by	[redacted]
Now, if we set the residual generator according to Corollary 12.1, then we have	[redacted]

As a result, Robt,DE delivers the maximal probability	[redacted]
while keeping the same FAR as given by (12.86). Remember that the prob- ability given in (12.87) is exactly the FDR given in Definition 12.2. In this context, we claim that the solution presented in this section, namely the uni- fied solution, also solves the FD systems design problem for stochastic systems (11.1)-(11.2), which is formulated as: given FAR (in the sense of Definition 12.1) find the residual generator,L and V, so that the FDR (in the sense of Definition 12.2) is maximized.	[redacted]
12.4.2 Application to minimizing FAR by a given FDR	[redacted]
The trade-off strategy proposed in Section 12.3 requires a threshold setting according to (12.59), which also fits the FDR in the sense of Definition 12.2,	[redacted]
For the computation of the associated FAR as defined in Definition 12.1, we can again use the estimation	[redacted]
Remember that the optimal residual generator Robt,FA ensures that	[redacted]
It results in a maximum probability which in turn means Ropt,FA offers the minimum bound for among all possible residual generators. In other words, Ropt,FA delivers a minimum FAR by a given FDR.	[redacted]
12.5 Notes and references	[redacted]

Although this chapter is less extensive in comparison with the other chapters, it is, in certain sense, the soul of this book. Di erent from the current way of solving the FDI problems in the context of robustness and sensitivity, as introduced in the previous chapters, the model-based FDI problems have been re-viewed in the context of FAR vs. FDR. Inspired by the interpretation of the concepts FAR and FDR in the statistical framework, we have

• introduced the concepts of FAR and FDR in the norm based context,

• defined SDF and SDFA and, based on them,

• formulated two trade-o problems: maximizing fault detectability by a given (allowable) FAR (PMax-SDF) and minimizing false alarm number by a given FDR (PMin-SDFA).

In this way, we have established a norm based framework for the analysis and design of observer-based FDI systems. It is important to notice that in this framework the four essential components of an observer-based FD system, the residual generator, residual evaluation function, the threshold and the decision logic, are taken into account by the problem formulations. This requires and also allows us to deal with the FDI system in an integrated manner. The integrated design distinguishes the design procedure proposed in this chapter significantly from the current strategies, where residual generation and evaluation are separately addressed. It has been demonstrated that the unified solution introduced in Chapter 7 also solves PMax-SDF, while the solution with inversing the

[redacted]

[redacted]

[redacted]

[redacted]

magnitude profile of the fault transfer function matrix is the one for PMin-SDFA. In the established norm based framework, a comparison study has further been undertaken. The results have verified, from the aspect of the trade-o FAR vs. FDR, that	[redacted]
• the unified solution leads to the maximum fault detectability under a given FAR and	[redacted]
• the ratio between the influences of the fault and the disturbances is the decisive factor for achieving the optimum performance and thus the influ- ence of the disturbance should be integrated into the reference model by designing a reference model based FD system.	[redacted]
One question may arise: why have we undertaken a so extensive study on the PUIDP in Chapter 6 and on the robustness issues in Chapter 7? To answer this question, we would like to call reader's attention to the result that the solution of the PUIDP is implicitly integrated into the general form of the unified solution (12.29). In fact, the solution of the PUIDP gives a factorization in the form of (7.305), which leads then to (12.29). Also, it should be pointed out that in the established norm framework, we have only addressed the FDI design problems under the assumption that the residual signals are evaluated in terms of the L2 norm. As outlined in Chapter 9, in practice also other kinds of signal norms are used for the purpose of residual evaluation. To study the FDI system design under these norms, the methods and tools introduced in Chapter 7 are	[redacted]

very helpful. As additional future work we would like to mention that an "LMI version" of the unified solution would help us to transfer the results achieved in this chapter to solving FDI problems met in dealing with other types of systems. In Section 12.4, we have briefly discussed the possible application of the proposed approaches to the stochastic systems. It would be also a promising topic for the future investigation. A useful tool to deal with such problems e ciently is the optimal selection of parity matrices presented in Section 7.5, which builds a link to the GLR technique.	[redacted]
A part of the results in this chapter has been provisionally reported in [31].	[redacted]
Fault isolation schemes	[redacted]
Fault isolation is one of the central tasks of a fault diagnosis system, a task that can become, by many practical applications, a real challenge for the system designer. Generally speaking, fault isolation is a signal processing process aiming at gaining information about the location of the faults occurred in the process under consideration. Evidently, the complexity of such a signal processing process strongly depends on	[redacted]
• the number of the possible faults,	[redacted]

• the possible distribution of the faults in the process under consideration,	[redacted]
• the characteristic features of each fault and	[redacted]
• the available information about the possible faults.	[redacted]
Correspondingly, the fault isolation problems will be solved step by step at different stages of a model-based fault diagnosis system. Depending on the number of the faults, their distribution and the fault isolation logic adopted in the decision unit, the residual generator should be so designed that the generated residual vector delivers the first clustering of the faults, which, in accordance with the fault isolation logic, divides the faults into a number of sets. At the residual evaluation stage, the characteristic features of the faults are then analyzed by using signal processing techniques based on the available information of the faults. As results, a further classification of the faults is achieved, and on its basis a decision about the location of the occurred faults is finally made. If the number of the faults is limited and their distribution is well structured, a fault isolation may become possible without a complex residual evaluation.	[redacted]
The main objective of this chapter is to present a number of widely used approaches for the purpose of fault isolation. Our focus is on the residual generation, as shown in Fig.13.1. We will first describe the basic principle, and then show the limitation of the fault isolation schemes which only rely on residual generators and without considering the characteristic features of the faults and thus without the application of special signal processing techniques for the residual	[redacted]

evaluation, and finally present and compare di erent observer- based fault isolation approaches.	[redacted]
13.1 Essentials	[redacted]
In this section, we first study the so-called perfect fault isolation (PFIs) problem formulated as: given system model with the fault vector f(p)ϵRkf, find a (linear) residual generator such that each component of the residual vecto r(p)ϵR¬¬¬¬¬kf corresponds to a fault defined by a component of the fault vector f(p). We do this for two reasons: by solving the PFIs problem	[redacted]
	[redacted]
• the role and, above all, the limitation of a residual generator for the purpose of fault isolation can be readily demonstrated and	• vai trò, và trên hết, hạn chế của khối phát tín hiệu dư trong cô lập lỗi có thể dễ dàng chứng minh được và
• the reader can get a deep insight into the underlying idea and basic principle of designing a residual generator for the purpose of fault isolation.	• người đọc sẽ hiểu sâu hơn các ý tưởng và nguyên lý cơ bản trong thiết kế khối phát tín hiệu dư nhằm cô lập lỗi.
On this basis, we will then present some approaches to the solution of the PFIs problem.	Trên cơ sở này, chúng tôi sẽ trình bày một số phương pháp giải bài toán PFIs.
13.1.1 Existence conditions for a perfect fault isolation	13.1.1 Điều kiện tồn tại đối với cô lập lỗi hoàn hảo
In order to study the existence conditions for a PFIs, we consider again the general form of the dynamics of the residual generator derived in Chapter 5	Để nghiên cứu các điều kiện tồn tại trong một PFIs, chúng ta lại xét dạng động học tổng quát của khối phát tín hiệu dư được rút ra ở Chương 5