Chương 4

Trường tĩnh điện trong vật chất

Bài tập 4.1

$E = V/x = 500/10^{-3} = 5\times 10^{5}$. Bảng 4.1: $\alpha/4\pi\varepsilon_0 = 0.66\times 10^{-30}$, vì vậy $\alpha = 4\pi(8.85\times 10^{-12})(0.66\times 10^{-30}) = 7.34\times 10^{-41}$.

$p = \alpha E = ed \Rightarrow d = \alpha E/e = (7.34\times 10^{-41})(5\times 10^{5})/(1.6\times 10^{-19}) = 2.29\times 10^{-16}$ m.

$d/R = (2.29\times 10^{-16})/(0.5\times 10^{-10}) = \boxed{4.6\times 10^{-6}}$. Để ion hóa, chẳng hạn d = R. Thì $R = \alpha E/e = \alpha V/ex \Rightarrow V = R\,ex/\alpha = (0.5\text{x}10^{-10})(1.6\text{x}10^{-19})(10^{-3})/(7.34\text{x}10^{-41}) = \boxed{10^{8}V.}$

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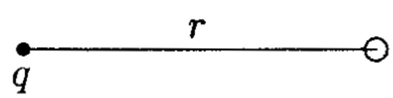

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$\theta = \pi$ [redacted]

$=$ [redacted]

$$\boxed{F = 2\alpha\left(\frac{q}{4\pi\epsilon_0}\right)^2 \frac{1}{r^3}}$$

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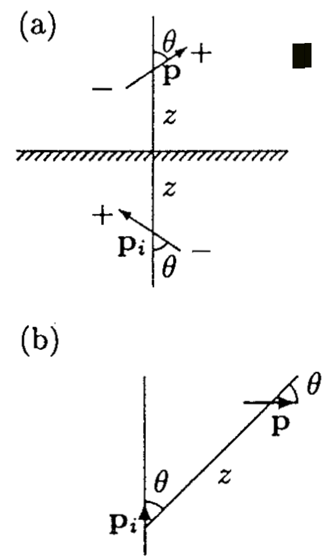
$\theta = \pi/$ [redacted] $=$ [redacted]

[redacted] = $\boxed{\dfrac{[redacted]}{[redacted]\pi\varepsilon[redacted]}}$ [redacted]

[redacted] $\theta = \pi$ [redacted] $E_2 = \frac{p_2}{4\pi\epsilon_0 r^3}(-2\hat{\boldsymbol{r}})$ [redacted]

[redacted] = $\boxed{\dfrac{[redacted]}{[redacted]\pi\varepsilon[redacted]}}$ [redacted]



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$$
\begin{aligned}
\mathrm{N} &= \mathbf{p} \times \mathbf{E}_i = \frac{p^2}{4\pi\epsilon_0(2z)^3}\left[(\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}) \times (2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})\right] \\
&= \frac{p^2}{4\pi\epsilon_0(2z)^3}\left[\cos\theta\sin\theta\,\hat{\boldsymbol{\phi}} + 2\sin\theta\cos\theta(-\,\hat{\boldsymbol{\phi}})\right] \\
&= \frac{p^2\sin\theta\cos\theta}{4\pi\epsilon_0(2z)^3}(-\hat{\boldsymbol{\phi}}) \quad \text{(out of the page).}
\end{aligned}
$$

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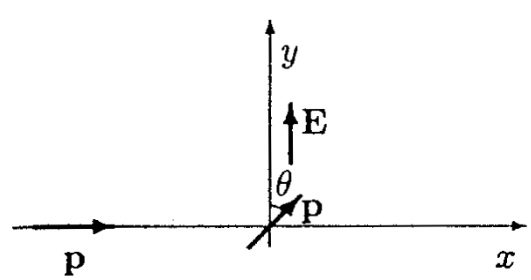

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∂ [redacted] $\pi\varepsilon$ [redacted]

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ρ[redacted] = [redacted]

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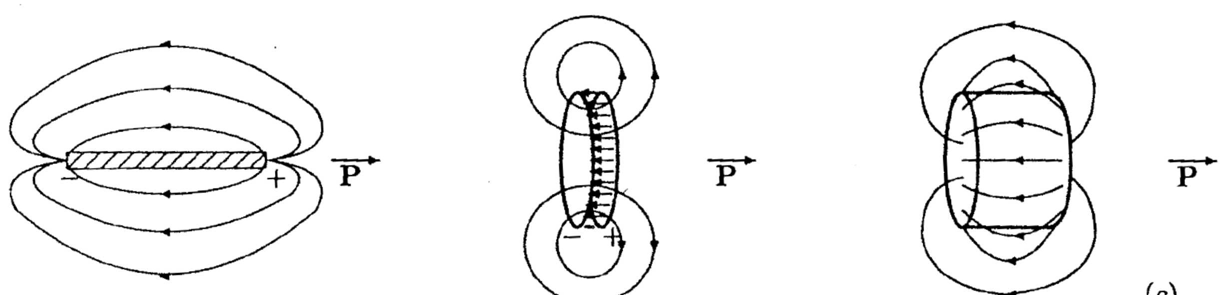

(a) Like a dipole (b) Like a parallel-plate capacitor (c)

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[redacted] $\int$ [redacted] $=$ [redacted] $\left\{ \begin{array}{l} \frac{[redacted]\pi\varepsilon}{} \; \pi \; [redacted] \\ \frac{[redacted]\pi\varepsilon}{} \; \pi \; [redacted] \end{array} \right.$ [redacted] $\theta = \left\{ \begin{array}{l} [redacted] \\ [redacted] \end{array} \right.$ [redacted]

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[redacted] $\pm\rho$ [redacted]

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[redacted] $= \rho/$ [redacted] ε [redacted]

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[redacted] $=$ [redacted] ε [redacted]

[redacted] $= \frac{\rho [redacted]}{[redacted]\varepsilon}$ [redacted]

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σ [redacted] $=$ [redacted] $= \{ +$ [redacted] $-$ [redacted]

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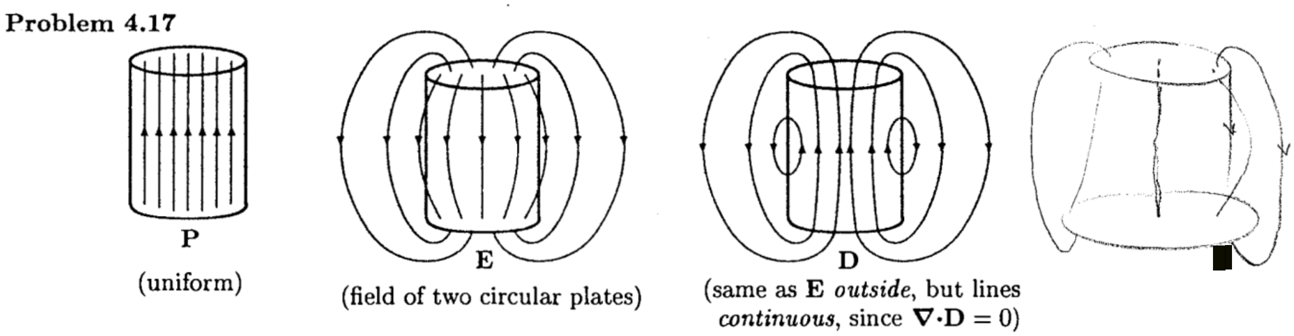

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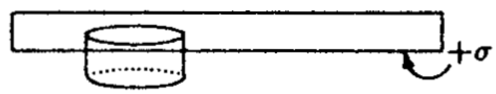

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ε [redacted]

[redacted] σ [redacted]

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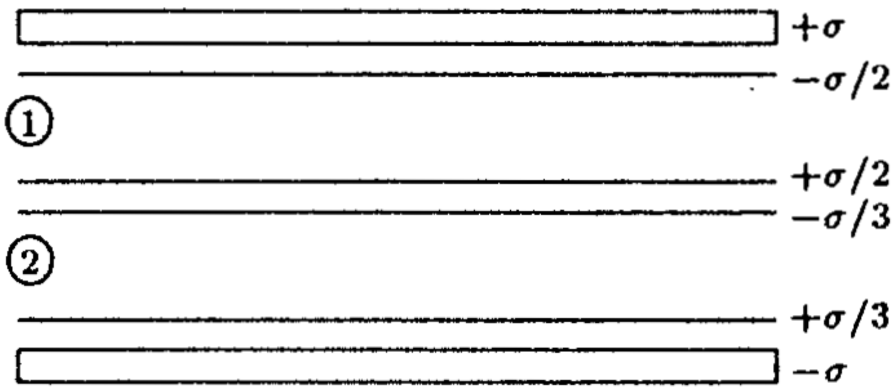

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[redacted] $= \sigma/\varepsilon$ [redacted] $= \sigma/\varepsilon$ [redacted]

[redacted] $= \frac{\sigma}{\varepsilon}$ [redacted] ε [redacted]

[redacted] $=$ [redacted] $=$ [redacted]

[redacted] $\sigma = \varepsilon$ [redacted]

[redacted] σ [redacted]

σ [redacted] σ [redacted]

$\Rightarrow$ [redacted] $=$ [redacted]

[redacted] $=$ [redacted] $> $ [redacted]

	$\mathbf{E}$	$\mathbf{D}$	$\mathbf{P}$	σ_b (top surface)	σ_f (top plate)
(a) air	$\frac{2\epsilon_r}{(\epsilon_r+1)}\frac{V}{d}\hat{\mathbf{x}}$	$\frac{2\epsilon_r}{(\epsilon_r+1)}\frac{\epsilon_0 V}{d}\hat{\mathbf{x}}$	0	0	$\frac{2\epsilon_r}{(\epsilon_r+1)}\frac{V}{d}$
(a) dielectric	$\frac{2}{(\epsilon_r+1)}\frac{V}{d}\hat{\mathbf{x}}$	$\frac{2\epsilon_r}{(\epsilon_r+1)}\frac{\epsilon_0 V}{d}\hat{\mathbf{x}}$	$\frac{2(\epsilon_r-1)}{(\epsilon_r+1)}\frac{\epsilon_0 V}{d}\hat{\mathbf{x}}$	$-\frac{2(\epsilon_r-1)}{(\epsilon_r+1)}\frac{\epsilon_0 V}{d}$	—
(b) air	$\frac{V}{d}\hat{\mathbf{x}}$	$\frac{\epsilon_0 V}{d}\hat{\mathbf{x}}$	0	0	$\frac{\epsilon_0 V}{d}$ (left)
(b) dielectric	$\frac{V}{d}\hat{\mathbf{x}}$	$\epsilon_r\frac{\epsilon_0 V}{d}\hat{\mathbf{x}}$	$(\epsilon_r-1)\frac{\epsilon_0 V}{d}\hat{\mathbf{x}}$	$-(\epsilon_r-1)\frac{\epsilon_0 V}{d}$	$\epsilon_r\frac{\epsilon_0 V}{d}$ (right)

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$\int$ [redacted]

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$= \frac{Q}{}$ [redacted] $\left[\ln\left(\frac{b}{a}\right)+\frac{\varepsilon}{\varepsilon}\ln\left(\frac{c}{b}\right)\right]$ [redacted]

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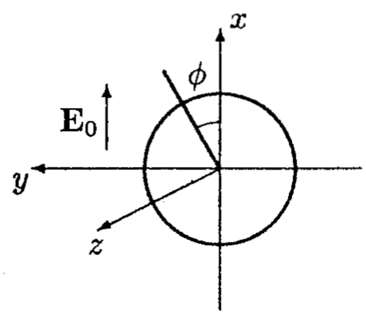

[redacted] $\varepsilon \frac{\partial}{\partial}$ [redacted]

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$\varepsilon \sum$ [redacted]

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$\sum^{\infty}$ [redacted] $=$ [redacted]

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[redacted] $\Rightarrow -$ [redacted] $\sum$ [redacted]

[redacted] $\Rightarrow \varepsilon \sum$ [redacted]

[redacted] $\Rightarrow$ [redacted]

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[redacted] $-$ [redacted] [redacted]

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[redacted] $=$ [redacted] $\left(r - \frac{[redacted]}{[redacted]}\right)\cos\theta$ [redacted]

$$\boxed{[redacted]\left\{ [redacted] \sin\theta [redacted] \right\}}$$ [redacted]

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[redacted] σ [redacted] ε [redacted] $\left[[redacted] - \frac{\sigma [redacted]}{[redacted]} \right]$ [redacted] σ [redacted]

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[redacted] $= \frac{[redacted]}{[redacted]\pi\varepsilon [redacted]}$ [redacted]

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$= \frac{[redacted]}{8\pi\varepsilon [redacted]}$ [redacted] $\left([redacted] + \frac{\chi [redacted]}{b} \right)$ [redacted]

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[redacted]

[redacted]

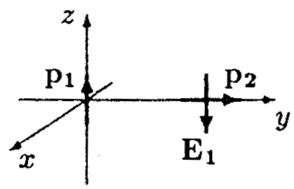
$$
\begin{aligned}
W_{r>R} &= \frac{\epsilon_0}{2}\left(\frac{R^3 P}{3\epsilon_0}\right)^2 \int \frac{1}{r^6}\left(4\cos^2\theta+\sin^2\theta\right) r^2 \sin\theta\, dr\, d\theta\, d\phi \\
&= \frac{(R^3P)^2}{18\epsilon_0} 2\pi \int_0^\pi \left(1+3\cos^2\theta\right)\sin\theta\, d\theta \int_R^\infty \frac{1}{r^4}\, dr = \frac{\pi (R^3P)^2}{9\epsilon_0}\left(-\cos\theta-\cos^3\theta\right)\Big|_0^\pi \left(-\frac{1}{3r^3}\right)\Big|_R^\infty \\
&= \frac{\pi (R^3P)^2}{9\epsilon_0}\left(\frac{4}{3R^3}\right) = \frac{4\pi R^3 P^2}{27\epsilon_0}.
\end{aligned}
$$

[redacted] $\boldsymbol{D} = \epsilon_0 \boldsymbol{E}$ [redacted]

[redacted] $\Rightarrow$ [redacted]

$$h = \frac{\epsilon_0 \chi_e V^2}{\rho(b^2 - a^2) g \ln(b/a)}.$$

[redacted] $\Rightarrow$ [redacted]



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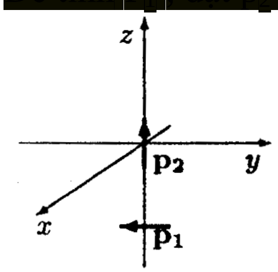

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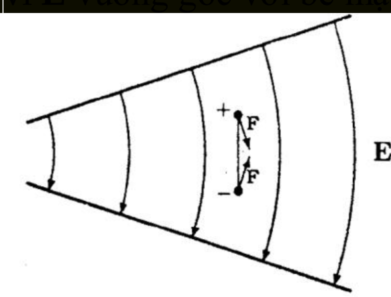

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[redacted] θ [redacted] ε [redacted] θ [redacted] ε [redacted]

[redacted] θ [redacted] ε [redacted] $>$ [redacted] θ [redacted] ε [redacted]

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[redacted] p [redacted] ε [redacted]

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[redacted] $\sum^{\infty}$ [redacted] $\cos\theta$ [redacted]

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[redacted] θ [redacted] $\pi\varepsilon$ [redacted] ε [redacted] $\cos\theta$ [redacted] $+\sum^{\infty}$ [redacted] $\cos\theta$ [redacted]

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[redacted] $\Rightarrow$ [redacted] $=$ [redacted] hoặc [redacted] $=$ [redacted] $+$ [redacted] $\pi\varepsilon$ [redacted] ε [redacted]

$\frac{\partial V}{\partial}$ [redacted] ∂V [redacted] $= -\sum$ [redacted] $\cos\theta$ [redacted] $\cos\theta$ [redacted] ε [redacted] σ [redacted]

$= -$ [redacted]

$-$ [redacted]

$$-2\frac{B_1}{R^3}+\frac{1}{4\pi\varepsilon_0}\frac{2p}{\varepsilon_r R^3}-A_1=\chi_e\left(-\frac{1}{4\pi\varepsilon_0}\frac{2p}{\varepsilon_r R^3}+A_1\right)-B_1+\frac{p}{4\pi\varepsilon_0\varepsilon_r}-\frac{A_1R^3}{2}=-\frac{1}{4\pi\varepsilon_0}\frac{\chi_e p}{\varepsilon_r}+\chi_e\frac{A_1R^3}{2}$$ [redacted]

$$-\frac{p}{4\pi\varepsilon_0\varepsilon_r}-A_1R^3+\frac{p}{4\pi\varepsilon_0\varepsilon_r}-\frac{A_1R^3}{2}=-\frac{1}{4\pi\varepsilon_0}\frac{\chi_e p}{\varepsilon_r}+\chi_e\frac{A_1R^3}{2}\Rightarrow\frac{A_1R^3}{2}\left(3+\chi_e\right)=\frac{1}{4\pi\varepsilon_0}\frac{\chi_e p}{\varepsilon_r}$$

$$\Rightarrow A_1=\frac{1}{4\pi\varepsilon_0}\frac{2\chi_e p}{R^3\varepsilon_r\left(3+\chi_e\right)}=\frac{1}{4\pi\varepsilon_0}\frac{2\left(\varepsilon_r-1\right)p}{R^3\varepsilon_r\left(\varepsilon_r+2\right)};B_1=\frac{p}{4\pi\varepsilon_0\varepsilon_r}\left[1+\frac{2\left(\varepsilon_r-1\right)}{\left(\varepsilon_r+2\right)}\right]=\frac{p}{4\pi\varepsilon_0\varepsilon_r}\frac{3\varepsilon_r}{\varepsilon_r+2}.$$ [redacted]

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σ [redacted] σ = ε [redacted]

$$\sigma = \begin{cases} \varepsilon \text{[redacted]} & \text{[redacted]} \\ \varepsilon \text{[redacted]} + \chi \text{[redacted]} & \text{[redacted]} \end{cases}$$

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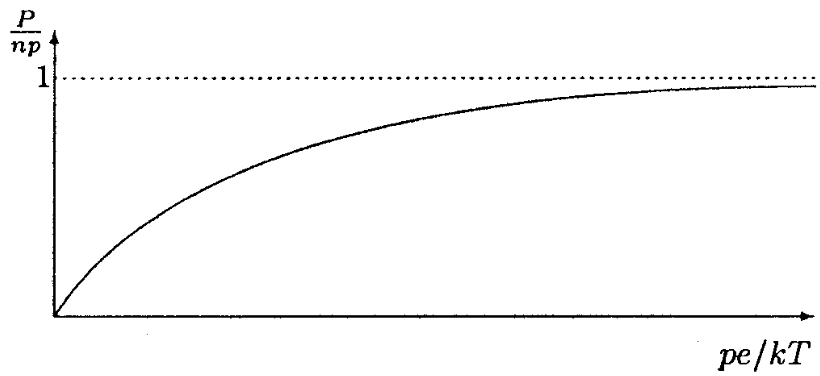

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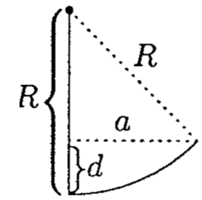

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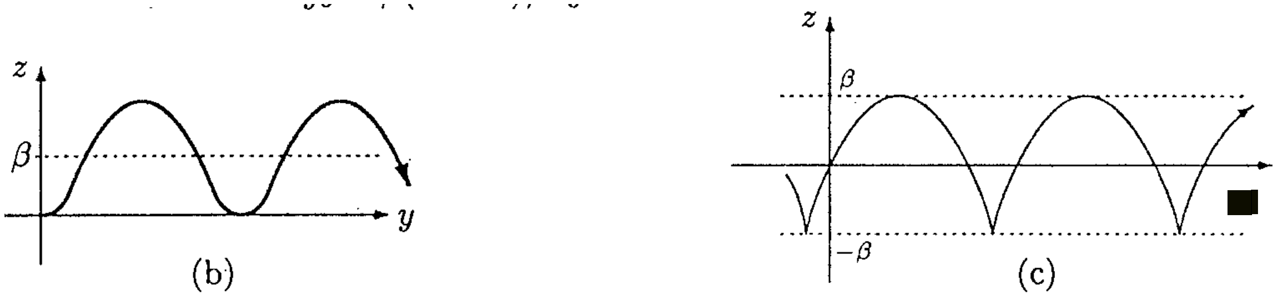

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[redacted] $B =$ [redacted]

[redacted] θ [redacted] $= \frac{\pi}{[redacted]}$ [redacted]

[redacted] $\rightarrow$ [redacted]

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$B =$ [redacted]

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[redacted] $\frac{\mu}{\pi}$ [redacted]

[redacted] $\frac{\mu}{[redacted]}$ [redacted]

[redacted] $= \frac{\mu}{[redacted]}\left([redacted] + \frac{[redacted]}{\pi}\right)$ [redacted]

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$B =$ [redacted] $B =$ [redacted]

[redacted] $\frac{\mu}{[redacted]}$ [redacted]

[redacted] μ [redacted] $/$ [redacted] π [redacted]

[redacted]

[redacted] $= -\frac{\mu}{[redacted]\pi} \int$ [redacted]

[redacted] $= -\frac{\mu}{2\sqrt{3}\pi}$ [redacted] $+$ [redacted]

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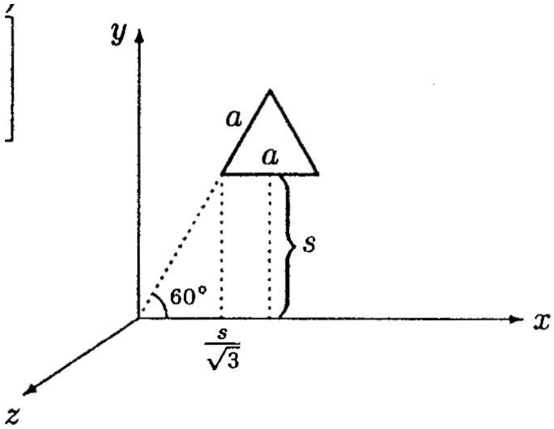

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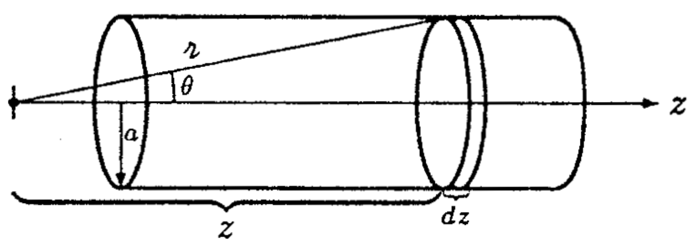

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[redacted] θ [redacted] $\theta = \pi$ [redacted]

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[redacted] $= \frac{\mu}{}$ [redacted] π $\frac{\lambda}{}$ [redacted]

[redacted] $= \frac{[redacted]}{[redacted]\pi\varepsilon_{[redacted]}} \frac{\lambda}{[redacted]}$ [redacted]

[redacted] $= \frac{[redacted]}{[redacted]\pi\varepsilon_{[redacted]}} \frac{\lambda^{[redacted]}}{[redacted]}$ [redacted] $\mu_{[redacted]} = \frac{[redacted]}{\varepsilon_{[redacted]}}$ [redacted] $\boxed{[redacted] = \frac{[redacted]}{\sqrt{\varepsilon_{[redacted]}}}}$ [redacted]

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[redacted] $\oint$ [redacted] $= \begin{cases} [redacted] \\ \frac{\mu_{[redacted]}}{[redacted]\pi s}\phi [redacted] \end{cases}$ [redacted]

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[redacted] $= \frac{[redacted]}{[redacted]}$ [redacted] $= \begin{cases} \frac{\mu_{[redacted]}}{[redacted]\pi [redacted]}\phi [redacted] \\ \frac{\mu_{[redacted]}}{[redacted]\pi [redacted]}\phi [redacted] \end{cases}$ [redacted]

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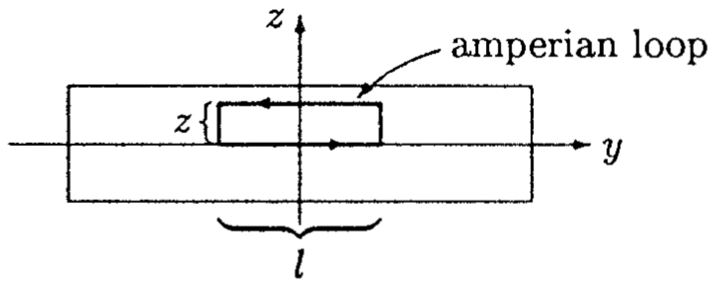

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[redacted] $\int$ [redacted]

[redacted] $\mu_0 \sigma \upsilon / 2$ [redacted]

[redacted] $\sigma/$ [redacted] ε [redacted]

[redacted] μ [redacted]

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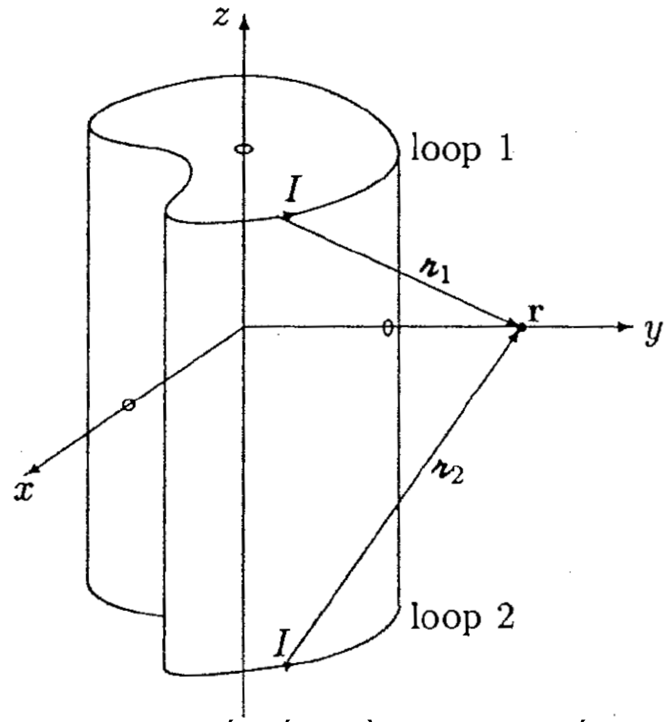
[redacted] $= -$ [redacted]

[redacted] $= \frac{\mu}{}$ [redacted] π [redacted]



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[redacted] $\rho =$ [redacted]

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$\rho =$ [redacted] $\boxed{\text{[redacted]}}$ [redacted]

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[redacted] $= \frac{[redacted]}{2\pi}$ [redacted] $= \frac{[redacted]}{2\pi} = \boxed{\text{[redacted]}}$ [redacted]

[redacted] $= \frac{[redacted]}{[redacted]\pi\varepsilon_{[redacted]}} \frac{\lambda}{[redacted]}$ [redacted]

[redacted] $= \frac{[redacted]}{[redacted]} = \frac{[redacted]}{[redacted]} = \left(\frac{[redacted]}{[redacted]} \right)^{[redacted]} = \boxed{\text{[redacted]}}$ [redacted]

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Bài tập 5.21

[redacted] $\boxed{\nabla\cdot [redacted] = \alpha_{[redacted]}\rho_{[redacted]}}$ [redacted] $\rho_{[redacted]}$ [redacted] $\alpha_{[redacted]}$ [redacted]

[redacted] ϵ_0 [redacted] $\mu_{[redacted]}$ [redacted] $\boxed{\nabla\times [redacted]}$ [redacted]

[redacted] $\beta_{[redacted]}$ [redacted]

[redacted] $\rho_{[redacted]}$ [redacted]

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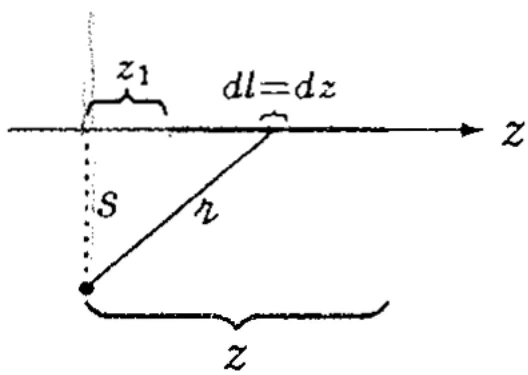
[redacted] α_0 [redacted]



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$$B = \nabla \times A = -\frac{\partial A}{\partial s}\hat{\phi} = -\frac{\mu_0 I}{4\pi}\left[\frac{1}{z_2+\sqrt{(z_2)^2+s^2}}\frac{s}{\sqrt{(z_2)^2+s^2}} - \frac{1}{z_1+\sqrt{(z_1)^2+s^2}}\frac{s}{\sqrt{(z_1)^2+s^2}}\right]\hat{\phi}$$

$$= -\frac{\mu_0 Is}{4\pi}\left[\frac{z_2-\sqrt{(z_2)^2+s^2}}{(z_2)^2-(z_2)^2+s^2}\frac{1}{\sqrt{(z_2)^2+s^2}} - \frac{z_2-\sqrt{(z_2)^2+s^2}}{(z_2)^2-(z_2)^2+s^2}\frac{1}{\sqrt{(z_2)^2+s^2}} - \frac{z_1-\sqrt{(z_1)^2+s^2}}{(z_1)^2-(z_1)^2+s^2}\frac{1}{\sqrt{(z_1)^2+s^2}}\right]\hat{\phi}$$

$$= -\frac{\mu_0 Is}{4\pi}\left(-\frac{1}{s^2}\right)\left[\frac{z_2}{\sqrt{(z_2)^2+s^2}} - 1 - \frac{z_1}{\sqrt{(z_1)^2+s^2}} + 1\right]\hat{\phi} = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{(z_2)^2+s^2}} - \frac{z_1}{\sqrt{(z_1)^2+s^2}}\right]\hat{\phi},$$

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[redacted]

[redacted] $= \frac{1}{\mu}\left[-\frac{\partial}{\partial s}\left(\frac{k}{s}\right)\right]$ [redacted]

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$\nabla \cdot$ [redacted] $\Big[$ [redacted] $\Big] =$ [redacted] $\nabla \times$ [redacted] $=$ [redacted] $\nabla \times$ [redacted] $=$ [redacted]

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[redacted] $=$ [redacted] $\nabla \cdot$ [redacted] $=$ [redacted] $\nabla \cdot$ [redacted] $=$ [redacted]

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$\nabla \times$ [redacted]

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[redacted] $=$ [redacted] $\nabla \cdot$ [redacted] $=$ [redacted] $\nabla \times$ [redacted]

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[redacted] $= \frac{\mu}{2\pi}$ [redacted] $\frac{\partial}{\partial s} =$ [redacted]

[redacted] $-\frac{\mu}{2\pi}$ [redacted] $=$ [redacted] $\frac{\mu}{4\pi}$ [redacted]

[redacted]

$$\boxed{[\text{redacted}] = \begin{cases} -\frac{\mu}{2\pi}\,[\text{redacted}] & [\text{redacted}] \\ -\frac{\mu}{2\pi}\,[\text{redacted}] & [\text{redacted}] \end{cases}}$$

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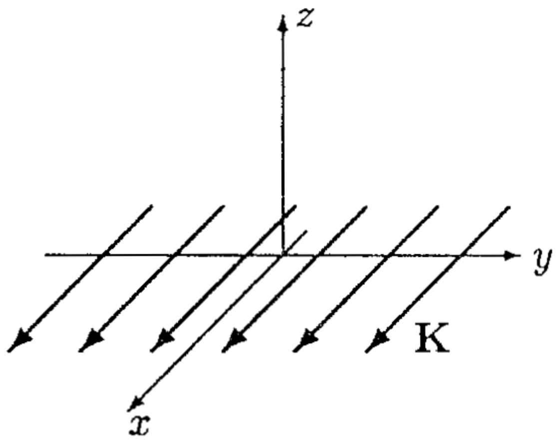

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[redacted] ∇ [redacted]

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μ [redacted] $\int$ [redacted] $-\int$ [redacted] $\Big]$ [redacted]

[redacted]

[redacted] $=\frac{\mu}{\ \pi}\phi$ [redacted] $\boxed{\ = -\frac{\mu\ \phi}{\ \pi}}$ [redacted]

$-\nabla$ [redacted] $\frac{\partial\phi}{\partial\phi}$ [redacted] ϕ [redacted] π [redacted]

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[redacted] $\sigma \to \rho$ [redacted] :

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[redacted] $=\Bigg($ [redacted] $\Bigg)$ [redacted]

[redacted] $\Bigg\}$ [redacted]

[redacted] $= \mu$ [redacted] $\omega\rho\Bigg[\Bigg($ [redacted] $\Bigg)$ [redacted] θ [redacted] $\Bigg]$ [redacted] $\rho = \frac{\ }{\ \pi}$ [redacted]

[redacted] $=$ $\boxed{\frac{\mu\ \omega}{\ }\Bigg[\Bigg(}$ [redacted] $\boxed{\theta\ \Bigg]}$ [redacted]

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[redacted] = [redacted]

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[redacted] μ [redacted] $\omega\sigma/$ [redacted] $\theta\phi$ [redacted]

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$\left(\frac{\partial}{\partial} \right. $ [redacted] $\left. \right)$ [redacted]

μ [redacted] $\frac{\partial}{\partial}$ [redacted] $= -\mu$ [redacted]

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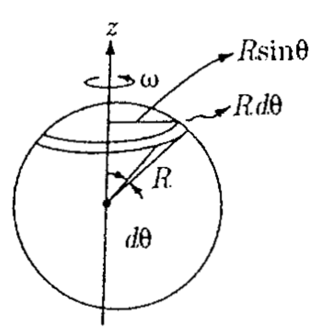

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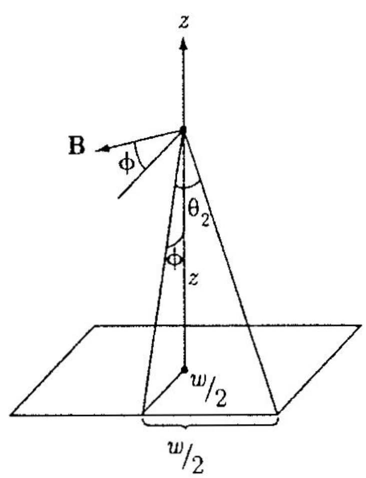

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[redacted] ρ [redacted] υ [redacted] $\int$ [redacted] $= \frac{\mu [redacted] \rho - \upsilon [redacted]}{[redacted]}\phi$ [redacted]

$\int$ [redacted] ε_0 [redacted]

[redacted] $\rho_+ + \rho$ [redacted] $\Rightarrow \rho_+ + \rho_- - \rho_-$ [redacted] $= \rho - \left(\frac{\upsilon [redacted]}{[redacted]}\right)$ [redacted]

[redacted] $\rho_+ = -\rho - \left([redacted] - \frac{\upsilon [redacted]}{[redacted]} \right)$ [redacted] $\frac{\rho_-}{\gamma}$ [redacted] $\rho_- = -\gamma [redacted] \rho_+$ [redacted]

[redacted] $\upsilon \ll c$ [redacted]

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[redacted] $\int$ [redacted] $\int$ [redacted]

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[redacted] $\int$ [redacted] $\frac{q}{[redacted]\pi}\int$ [redacted] $\boxed{[redacted] = -\frac{[redacted]}{[redacted]\pi}\phi}$ [redacted] $\phi = \int$ [redacted]

[redacted] $\phi =$ [redacted] $=$ [redacted]

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[redacted] $\theta\phi$ [redacted] $\Rightarrow \phi =$ [redacted]

[redacted] θ [redacted]

[redacted] $\upsilon^2 -$ [redacted] $\theta =$ [redacted] $\phi =$ [redacted]

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[redacted] $\phi =$ [redacted] θ [redacted] $= -$ [redacted] θ [redacted]

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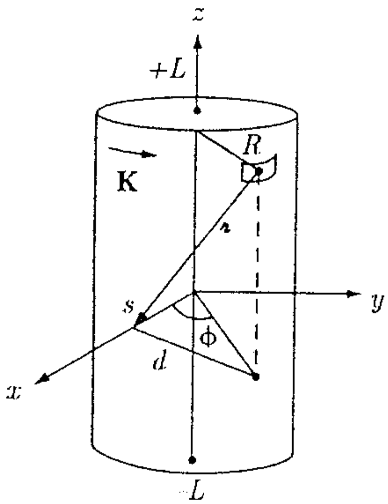
[redacted] $\boldsymbol{r} = (s, 0, 0)$ [redacted]

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{(\mathbf{K}\times\hat{\boldsymbol{\imath}})}{\imath^2}\,da; \quad da = R\,d\phi\,dz; \quad \mathbf{K} = K\hat{\boldsymbol{\phi}} = K(-\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}); \boldsymbol{\imath} = (s - R\cos\phi)\,\hat{\mathbf{x}} - R\sin\phi\,\hat{\mathbf{y}} - z\,\hat{\mathbf{z}}.$$

$$\mathbf{K}\times\boldsymbol{\imath} = K\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ -\sin\phi & \cos\phi & 0 \\ (s - R\cos\phi) & (-R\sin\phi) & (-z) \end{vmatrix} = K\left[(-z\cos\phi)\,\hat{\mathbf{x}} + (-z\sin\phi)\,\hat{\mathbf{y}} + (R - s\cos\phi)\,\hat{\mathbf{z}}\right];$$

$$\imath^2 = z^2 + R^2 + s^2 - 2Rs\cos\phi.$$



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$$
\begin{aligned}
B_z &= \frac{\mu_0}{4\pi} K R \int \frac{(R - s\cos\phi)}{(z^2 + R^2 + s^2 - 2Rs\cos\phi)^{3/2}}\, d\phi\, dz \\
&= \frac{\mu_0 K R}{4\pi} \int_0^{2\pi} (R - s\cos\phi) \left\{ \int_{-\infty}^{\infty} \frac{dz}{(z^2 + d^2)^{3/2}} \right\} d\phi,
\end{aligned}
$$

where $d^2 \equiv R^2 + s^2 - 2Rs\cos\phi$. Now $\int_{-\infty}^{\infty} \frac{dz}{(z^2+d^2)^{3/2}} = \left. \frac{2z}{d^2\sqrt{z^2+d^2}} \right|_0^{\infty} = \frac{2}{d^2}$.

$$
\begin{aligned}
&= \frac{\mu_0 K R}{2\pi} \int_0^{2\pi} \frac{(R - s\cos\phi)}{(R^2 + s^2 - 2Rs\cos\phi)}\, d\phi; \quad (R - s\cos\phi) = \frac{1}{2R}\left[(R^2 - s^2) + (R^2 + s^2 - 2Rs\cos\phi)\right]. \\
&= \frac{\mu_0 K}{4\pi} \left[(R^2 - s^2) \int_0^{2\pi} \frac{d\phi}{(R^2 + s^2 - 2Rs\cos\phi)} + \int_0^{2\pi} d\phi \right].
\end{aligned}
$$

$$
\begin{aligned}
\int_0^{2\pi} \frac{d\phi}{a + b\cos\phi} &= 2\int_0^{\pi} \frac{d\phi}{a + b\cos\phi} = \frac{4}{\sqrt{a^2 - b^2}} \tan^{-1}\left[\frac{\sqrt{a^2 - b^2}\tan(\phi/2)}{a + b} \right]\Bigg|_0^{\pi} \\
&= \frac{4}{\sqrt{a^2 - b^2}} \tan^{-1}\left[\frac{\sqrt{a^2 - b^2}\tan(\pi/2)}{a + b} \right] = \frac{4}{\sqrt{a^2 - b^2}}\left(\frac{\pi}{2}\right) = \frac{2\pi}{\sqrt{a^2 - b^2}}. \text{ Here } a = R^2 + s^2,
\end{aligned}
$$

$b = -2Rs$, so $a^2 - b^2 = R^4 + 2R^2s^2 + s^4 - 4R^2s^2 = R^4 - 2R^2s^2 + s^4 = (R^2 - s^2)^2$; $\sqrt{a^2 - b^2} = |R^2 - d^2|$.

$$
B_z = \frac{\mu_0 K}{4\pi}\left[\frac{(R^2 - s^2)}{|R^2 - s^2|} 2\pi + 2\pi \right] = \frac{\mu_0 K}{2}\left(\frac{R^2 - s^2}{|R^2 - s^2|} + 1 \right).
$$

[redacted]

$$
B_z = \frac{\mu_0 K}{2}(1+1) = \mu_0 K.
$$

[redacted]

$$B_z = \frac{\mu_0 K}{2}(-1+1) = 0$$

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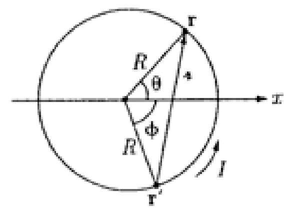
$$\boxed{\mathbf{B} = \mu_0 n I\,\hat{\mathbf{z}}(\text{inside}),\ \text{and } 0(\text{outside})}$$

[redacted] $\boldsymbol{r}' = Rcos\phi\hat{\boldsymbol{x}} - Rsin\phi\hat{\boldsymbol{y}}$ [redacted] $\boldsymbol{r} = Rcos\theta\hat{\boldsymbol{x}} + Rsin\theta\hat{\boldsymbol{y}}$ [redacted]

$\boldsymbol{\mathscr{r}} = R\left[(\cos\theta - \cos\phi)\,\hat{\mathbf{x}} + (\sin\theta + \sin\phi)\,\hat{\mathbf{y}}\right]$ and $d\mathbf{l} = R\sin\phi\, d\phi\,\hat{\mathbf{x}} + R\cos\phi\, d\phi\,\hat{\mathbf{y}} = R\,d\phi(\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}})$.



$$\begin{aligned}
d\mathbf{l}\times\boldsymbol{\mathscr{r}} &= R^2\,d\phi\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ \sin\phi & \cos\phi & 0\\ (\cos\theta-\cos\phi) & (\sin\theta+\sin\phi) & 0\end{vmatrix}\\
&= R^2(\sin\phi\sin\theta + \sin^2\phi - \cos\theta\cos\phi + \cos^2\phi)\,d\phi\,\hat{\mathbf{z}}\\
&= R^2(1+\sin\theta\sin\phi - \cos\theta\cos\phi)\,d\phi\,\hat{\mathbf{z}} = R^2\left[1-\cos(\theta+\phi)\right]\,d\phi\,\hat{\mathbf{z}}.\\
\mathbf{B} &= \frac{\mu_0 I}{4\pi}\int\frac{d\mathbf{l}\times\boldsymbol{\mathscr{r}}}{\mathscr{r}^3} = \frac{\mu_0 I}{4\pi}R^2\,\hat{\mathbf{z}}\int_0^{\pi}\frac{[1-\cos(\theta+\phi)]}{[2R^2-2R^2\cos(\theta+\phi)]^{3/2}}\,d\phi = \frac{\mu_0 I R^2}{4\pi(2R^2)^{3/2}}\,\hat{\mathbf{z}}\int_0^{\pi}\frac{d\phi}{\sqrt{1-\cos(\theta+\phi)}}\\
&= \frac{\mu_0 I}{8\sqrt{2}\pi R}\,\hat{\mathbf{z}}\int_0^{\pi}\frac{d\phi}{\sqrt{2}\sin[(\theta+\phi)/2]} = \frac{\mu_0 I}{16\pi R}\,\hat{\mathbf{z}}\left\{2\ln\left[\tan\left(\frac{\theta+\phi}{4}\right)\right]\right\}\Bigg|_0^{\pi} = \boxed{\frac{\mu_0 I}{8\pi R}\ln\left[\frac{\tan\left(\frac{\theta+\pi}{4}\right)}{\tan\left(\frac{\theta}{4}\right)}\right]\hat{\mathbf{z}}.}
\end{aligned}$$

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$$\boxed{\mathbf{B} = \frac{\mu_0 I R^2}{2}\left\{\frac{1}{[R^2+(d/2+z)^2]^{3/2}} + \frac{1}{[R^2+(d/2-z)^2]^{3/2}}\right\}.}$$

$$\begin{aligned}
\frac{\partial B}{\partial z} &= \frac{\mu_0 I R^2}{2}\left\{\frac{(-3/2)2(d/2+z)}{[R^2+(d/2+z)^2]^{5/2}} + \frac{(-3/2)2(d/2-z)(-1)}{[R^2+(d/2-z)^2]^{5/\text{[redacted]}}}\right\} \\
&= \frac{3\mu_0 I R^2}{2}\left\{\frac{-(d/2+z)}{[R^2+(d/2+z)^2]^{5/2}} + \frac{(d/2-z)}{[R^2+(d/2-z)^2]^{5/2}}\right\}.
\end{aligned}$$

$$\left.\frac{\partial B}{\partial z}\right|_{z=0} = \frac{3\mu_0 I R^2}{2}\left\{\frac{-d/2}{[R^2+(d/2)^2]^{5/2}} + \frac{d/2}{[R^2+(d/2)^2]^{5/2}}\right\} = 0. \checkmark$$

[redacted]

$$\begin{aligned}
\frac{\partial^2 B}{\partial z^2} &= \frac{3\mu_0 I R^2}{2}\left\{\frac{-1}{[R^2+(d/2+z)^2]^{5/2}} + \frac{-(d/2+z)(-5/2)2(d/2+z)}{[R^2+(d/2+z)^2]^{7/2}}\right. \\
&\quad \left. + \frac{-1}{[R^2+(d/2-z)^2]^{5/2}} + \frac{(d/2-z)(-5/2)2(d/2-z)(-1)}{[R^2+(d/2-z)^2]^{7/2}}\right\}.
\end{aligned}$$

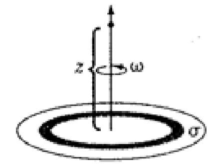
$$\begin{aligned}
\left.\frac{\partial^2 B}{\partial z^2}\right|_{z=0} &= \frac{3\mu_0 I R^2}{2}\left\{\frac{-2}{[R^2+(d/2)^2]^{5/2}} + \frac{2(5/2)2(d/2)^2 2}{[R^2+(d/2)^2]^{7/2}}\right\} = \frac{3\mu_0 I R^2}{[R^2+(d/2)^2]^{7/2}}\left(-R^2 - \frac{d^2}{4} + \frac{5d^2}{4}\right) \\
&= \frac{3\mu_0 I R^2}{[R^2+(d/2)^2]^{7/2}}\left(d^2 - R^2\right)
\end{aligned}$$

Bằng 0 nếu $\boxed{d = R,}$ trong trường hợp đó

$$B(0) = \frac{\mu_0 I R^2}{2}\left\{\frac{1}{[R^2+(R/2)^2]^{3/2}} + \frac{1}{[R^2+(R/2)^2]^{3/2}}\right\} = \mu_0 I R^2 \frac{1}{(5R^2/4)^{3/2}} = \boxed{\frac{8\mu_0 I}{5^{3/2} R}.}$$

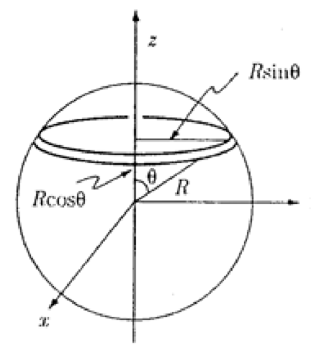
Bài tập 5.47

(a) Điện tích toàn phần trên vòng được tô là $dq = \sigma(2\pi r)\,dr$. Thời gian để quay một vòng là $dt = 2\pi/\omega$. Vì vậy dòng điện trong vòng là $I = \frac{aq}{dt} = \sigma\omega r\,dr$. Từ phương trình 5.38, từ trường của vòng này (đối với các điểm trên trục) là $dB = \frac{\mu_0}{2}\sigma\omega r \frac{r^2}{(r^2+z^2)^{3/2}}\,dr\,\hat{\mathbf{z}}$, và trường toàn phần của đĩa là



$$\begin{aligned}
\mathbf{B} &= \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{r^3\,dr}{(r^2+z^2)^{3/2}}\,\hat{\mathbf{z}}. \quad \text{Đặt } u \equiv r^2, \text{ so } du = 2r\,dr. \quad \text{Do đó} \\
&= \frac{\mu_0\sigma\omega}{4}\int_0^{R^2} \frac{u\,du}{(u+z^2)^{3/2}} = \frac{\mu_0\sigma\omega}{4}\left[2\left(\frac{u+2z^2}{\sqrt{u+z^2}}\right)\right]\Bigg|_0^{R^2} = \boxed{\frac{\mu_0\sigma\omega}{2}\left[\frac{(R^2+2z^2)}{\sqrt{R^2+z^2}} - 2z\right]\hat{\mathbf{z}}.}
\end{aligned}$$

(b) Cắt hình cầu thành những miếng có chiều dày t, và dùng (a). Ở đây $t = |d(R\cos\theta)| = R\sin\theta\,d\theta$; $\sigma \to \rho t = \rho R\sin\theta\,d\theta;\ R \to R\sin\theta;\ z \to z - R\cos\theta$. Trước hết viết lại số hạng trong các dấu ngoặc vuông:



$$\begin{aligned}
\left[\frac{(R^2+2z^2)}{\sqrt{R^2+z^2}} - 2z\right] &= \frac{2(R^2+z^2)}{\sqrt{R^2+z^2}} - \frac{R^2}{\sqrt{R^2+z^2}} - 2z \\
&= 2\left[\sqrt{R^2+z^2} - \frac{R^2/2}{\sqrt{R^2+z^2}} - z\right].
\end{aligned}$$

Nhưng $R^2 + z^2 \to R^2\sin^2\theta + (z^2 - 2Rz\cos\theta + R^2\cos^2\theta) = R^2 + z^2 - 2Rz\cos\theta$. Vì vậy

$$\begin{aligned}
B_z &= \frac{\mu_0\rho R\omega}{2} 2\int_0^\pi \sin\theta\,d\theta\left[\sqrt{R^2+z^2-2Rz\cos\theta} - \frac{(R^2/2)\sin^2\theta}{\sqrt{R^2+z^2-2Rz\cos\theta}} - (z - R\cos\theta)\right]. \\
&\quad \text{Let } u \equiv \cos\theta, \text{ so } du = -\sin\theta\,d\theta;\ \theta: 0 \to \pi \Rightarrow u: 1 \to -1;\ \sin^2\theta = 1 - u^2. \\
&= \mu_0\rho R\omega\int_{-1}^1\left[\sqrt{R^2+z^2-2Rzu} - \frac{(R^2/2)(1-u^2)}{\sqrt{R^2+z^2-2Rzu}} - z + Ru\right]du \\
&= \mu_0\rho R\omega\left[I_1 - \frac{R^2}{2}(I_2 - I_3) - I_4 + I_5\right].
\end{aligned}$$

$$\begin{aligned}
I_1 &= \int_{-1}^1 \sqrt{R^2+z^2-2Rzu}\,du = -\frac{1}{3Rz}\left(R^2+z^2-2Rzu\right)^{3/2}\Big|_{-1}^1 \\
&= -\frac{1}{3Rz}\left[\left(R^2+z^2-2Rz\right)^{3/2} - \left(R^2+z^2+2Rz\right)^{3/2}\right] = -\frac{1}{3Rz}\left[(z-R)^3 - (z+R)^3\right] \\
&= -\frac{1}{3Rz}\left(z^3 - 3z^2R + 3zR^2 - R^3 - z^3 - 3z^2R - 3zR^2 - R^3\right) = \frac{2}{3z}(3z^2 + R^2). \\
I_2 &= \int_{-1}^1 \frac{1}{\sqrt{R^2+z^2-2Rzu}}\,du = -\frac{1}{Rz}\sqrt{R^2+z^2-2Rzu}\Big|_{-1}^1 = -\frac{1}{Rz}\left[(z-R) - (z+R)\right] = \frac{2}{z}.
\end{aligned}$$

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$$F = -\frac{\mu_0}{4\pi} I_1 I_2 \left\{ \oint\oint \frac{\hat{r}}{r^2}\left(dL_1 . dL_2\right) - \oint dL_1 \oint \frac{\left(dL_2 . \hat{r}\right)}{r^2} \right\}$$

[redacted]

$r = (x_2 - x_1)\hat{x} + (y_2 - y_1)\hat{y} + (z_2 - z_1)\hat{z}$, vì vậy

$$\nabla_2(1/r) = \frac{\partial}{\partial x_2}\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{-1/2}\hat{x} + \frac{\partial}{\partial y_2}\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{-1/2}\hat{y} +$$

$$\frac{\partial}{\partial z_2}\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{-1/2}\hat{z}$$

$$= -\frac{(x_2 - x_1)}{r^3}\hat{x} - \frac{(y_2 - y_1)}{r^3}\hat{y} - \frac{(z_2 - z_1)}{r^3}\hat{z}$$

$$= -\frac{r}{r^3} = -\frac{\hat{r}}{r^2}$$

[redacted]

Bài tập 5.50

Phương trình Poisson (Pt. 2.24) có dạng $\nabla^2 V = -\frac{1}{\varepsilon_0}\rho$. Đối với các chất điện môi (không có điện tích tự do), $\rho_b = -\nabla . P$ (Pt.4.12), và thế năng cuối cùng là $V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{P(r')\hat{r}}{r^2}dt'$. Nói chung, $\rho = \varepsilon_0 \nabla$. E (định luật Gauss), vì vậy tương tự ta[redacted]ó P → $-\varepsilon_0 E$, và vì thế $V(r) = -\frac{1}{4\pi}\int \frac{E(r')\hat{r}}{r^2}dt'$. ĐLĐPCM

[có nhiều cách khác để thu được kết quả này. Chẳng hạn, dùng Pt. 1.100:

$\nabla . \left(\frac{\hat{r}}{r^2}\right) = -\nabla' . \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta(r) = 4\pi\delta^3(r - r')$,

$V(r) = \int V(r')\delta^3(r - r')dt' = -\frac{1}{4\pi}\int V(r')\nabla' . (\frac{\hat{r}}{r^2})dt' = \frac{1}{4\pi}\int \frac{\hat{r}}{r^2} . [\nabla' V(r')]dt' - \frac{1}{4\pi}\oint V(r')\frac{\hat{r}}{r^2} . da'$

(Pt. 1.59). nhưng $\nabla' V(r') = -E(r')$, và tích phân mặt→ 0 tại ∞, vì vậy $V(r) = -\frac{1}{4\pi}\int \frac{E(r') . \hat{r}}{r^2}dt'$, như trước. Bạn cũng có thể kiểm tra kết quả bằng cách tính toán gradient của nó – nhưng việc đó không dễ.]

Bài tập 5.51

(a) Đối với B đều, $\int_0^r (B\times dl) = B\times\int_0^r dl = B\times r \neq A = -\frac{1}{2}(B\times r)$

(b) $B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$, vì vậy $\oint B\times dl = \left(\frac{\mu_0 I}{2\pi a}\hat{s} - \frac{\mu_0 I}{2\pi b}\hat{s}\right)\omega = \frac{\mu_0 I\omega}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)\hat{s} \neq 0$

(c) $A = -r\times B\int_0^1 \lambda d\lambda = -\frac{1}{2}(r\times B)$

(d) $B = \frac{\mu_0 I}{2\pi s}\hat{\phi}, B(\lambda r) = \frac{\mu_0 I}{2\pi\lambda s}\hat{\phi}, A = -\frac{\mu_0 I}{2\pi s}(r\times\hat{\phi})\int_0^1 \lambda\frac{1}{\lambda}d\lambda = -\frac{\mu_0 I}{2\pi s}(r\times\hat{\phi})$. Nhưng ở đây r là vector từ gốc tọa độ—trong các tọa độ trụ $r = s\hat{s} + z\hat{z}$. Vì vậy $A = -\frac{\mu_0 I}{2\pi s}\left[s(\hat{s}\times\hat{\phi}) + z(\hat{z}\times\hat{\phi})\right]$, và $(\hat{s}\times\hat{\phi}) = \hat{z}, (\hat{z}\times\hat{\phi}) = -\hat{s}$. Vì vậy $A = \frac{\mu_0 I}{2\pi s}(z\hat{s} - s\hat{z})$.

Các ví dụ trong (c) và (d) ngẫu nhiên không phân kỳ, nhưng điều này không đúng trong trường hợp tổng quát. Đối với (đặt $L \equiv \int_0^1 \lambda B(\lambda r)d\lambda$, để ngắn gọn)

$\nabla.A = -\nabla.(r\times L) = -[L.(\nabla\times r) - r.(\nabla\times L)] = r.(\nabla\times L)$, và

$\nabla \times L = \int_0^1 \lambda[\nabla \times B(\lambda r)]d\lambda = \int_0^1 \lambda^2[\nabla_\lambda \times B(\lambda r)]d\lambda = \mu_0 \int_0^1 \lambda^2 J(\lambda r)d\lambda$, vì vậy

$\nabla . A = -\nabla .(r \times L) = -[L.(\nabla \times r) - r.(\nabla \times L)] = r.(\nabla \times L)$, và $\nabla \times L = \int_0^1 \lambda[\nabla \times B(\lambda r)]d\lambda = \int_0^1 \lambda^2[\nabla_\lambda \times B(\lambda r)]d\lambda = \mu_0 \int_0^1 \lambda^2 J(\lambda r)d\lambda$, vì vậy

$\nabla . A = \mu_0 r. \int_0^1 \lambda^2 J(\lambda r)d\lambda$, và nó triệt tiêu trong các vùng J = 0 (đó là lí do tại sao các ví dụ trong (c) và (d) không phân kỳ). Để tạo ra một phản ví dụ tường minh, chúng ta cần trường tại một điểm $J \neq 0$—chẳng hạn như, bên trong dây với dòng đồng đều.

Ở đây định luật ampe cho chúng ta $B 2\pi s = \mu_0 I_{enc} = \mu_0 I \pi s^2 \Rightarrow B = \frac{\mu_0 J}{2} s\hat{\phi}, so$

$$A = -r \times \int_0^1 \lambda \left(\frac{\mu_0 J}{2}\right) \lambda s \hat{\phi} d\lambda = -\frac{\mu_0 I}{6} s(r \times \hat{\phi}) = \frac{\mu_0 J s}{6}(z\hat{s} - s\hat{z}).$$

$$\nabla . A = \frac{\mu_0 J}{6}\left[\frac{1}{s}\frac{\partial}{\partial s}(s^{2z}) + \frac{\partial}{\partial z}(-s^2)\right] = \frac{\mu_0 J}{6}\left(\frac{1}{s} 2sz\right) = \frac{\mu_0 J z}{3} \neq 0$$

Kết luận: (ii) không tự động mang lại kết quả $\nabla . A = 0$.

Bài tập 5.52

(a) Khai thác tính tương tự với trường hợp điện:

$$E=\frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3(P.\hat{r})\hat{r}-P\right] \quad \text{(Pt . 3.104)} = -\nabla V \text{ , với } V=\frac{1}{4\pi\varepsilon_0}\frac{P.\hat{r}}{r^2} \text{(Pt .3.102).}$$

$$\mathbf{B}=\frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(m.\hat{r})\hat{r}-m\right] \quad \text{(Pt.5.5.87)} = -\nabla U \text{ , (Pt.5.56).}$$

Tất nhiên điều kiện ràng buộc là $P/\varepsilon_0 \to \mu_0 m : U(r)=\frac{\mu_0}{4\pi}\frac{m.\hat{r}}{r^2}$.

(b) So sánh các phương trình 5.67 và 5.85, momen lưỡng cực của vỏ là $m=(4\pi/3)\omega\sigma R^4\hat{z}$ (chúng ta cũng có được kết quả này trong bt 5.36). Dùng kết quả của (a) , thì, $U(r)=\frac{\mu_0\omega\sigma R^4}{3}\frac{\cos\theta}{r^2}$ khi $r>R$.

Bên trong vỏ, trường đều (Pt.5.38): $B=\frac{2}{3}\mu_0\sigma\omega R\hat{z}$, vì vậy $U(r)-\frac{2}{3}\mu_0\sigma\omega Rz$ + hằng số. Chúng ta cũng có thể chọn hằng số bằng không, vì vậy $U(r)=-\frac{2}{3}\mu_0\sigma\omega Rr\ \cos\theta$ khi $r<R$.

[chú ý rằng U(r) không liên tục tại bề mặt ($r = R$) : $U_{in}(R)--\frac{2}{3}\mu_0\sigma\omega R^2$ cos $\theta \neq U_{out}(R)=\frac{1}{3}\mu_0\sigma\omega R^2\cos\theta$. Như tôi đã cảnh báo với bạn ở trang 236 : nếu bạn nhất định đòi dùng các thế từ vô hướng, hãy tránh xa những nơi có dòng !]

(c)

$$B=\frac{\mu_0\omega Q}{4\pi R}\left[\left(1-\frac{3r^2}{5R^2}\right)\cos\theta\hat{r}-\left(1-\frac{6r^2}{5R^2}\right)\sin\theta\hat{\phi}\right]=-\nabla U=-\frac{\partial U}{\partial r}\hat{r}-\frac{1}{r}\frac{\partial U}{\partial\theta}\hat{\theta}-\frac{1}{r\sin\theta}\frac{\partial U}{\partial\phi}\hat{\phi}.$$

$$\frac{\partial U}{\partial\phi}=0\Rightarrow U(r,\theta,\phi)=U(r,\theta.)$$

$$\frac{1}{r}\frac{\partial U}{\partial\theta}=\left(\frac{\mu_0\omega Q}{4\pi R}\right)\left(1-\frac{6r^2}{5R^2}\right)\sin\theta\Rightarrow U(r,\theta)=-\left(\frac{\mu_0\omega Q}{4\pi R}\right)\left(1-\frac{6r^2}{5R^2}\right)r\cos\theta+f(r).$$

$$\frac{\partial U}{\partial r}=-\left(\frac{\mu_0\omega Q}{4\pi R}\right)\left(1-\frac{3r^2}{5R^2}\right)\cos\theta\Rightarrow U(r,\theta)=-\left(\frac{\mu_0\omega Q}{4\pi R}\right)\left(r-\frac{r^3}{5R^2}\right)\cos\theta+g(\theta).$$

[redacted]

$$-\left(\frac{\mu_0 \omega Q}{4\pi R}\right)\left(1-\frac{6r^2}{5R^2}\right) r\cos\theta + f(r) = -\left(\frac{\mu_0 \omega Q}{4\pi R}\right)\left(1-\frac{r^2}{5R^2}\right) r\cos\theta + g(\theta),$$

[redacted] $\left(\frac{\mu_0 \omega Q}{4\pi R^3}\right) r^3 \cos\theta + f(r) = g(\theta)$ [redacted]

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[redacted] ∇ [redacted] $=$ [redacted] $\nabla \times$ [redacted] $= \mu$ [redacted] ∇ [redacted] $=$ [redacted] $\nabla \times$ [redacted] $=$ [redacted] $\Rightarrow$ [redacted] $= \frac{\mu}{[redacted]\pi} \int$ [redacted] ∇ [redacted] $=$ [redacted]

$\nabla \times$ [redacted] $=$ [redacted] ∇ [redacted] $=$ [redacted] ∇ [redacted] $=$ [redacted] $\Rightarrow$ [redacted] $= \frac{[redacted]}{[redacted]\pi} \int$ [redacted]

(b) W sẽ tỉ lệ với B và với hai thừa số của r (vì lấy vi phân lần hai sẽ phục hồi B), vì vậy tôi sẽ thử hàm nào đó có dạng $w = \alpha r(r.B) + \beta r^2 B$, và thử xem tôi có thể chọn các hằng số α và β để $\nabla.W = 0$ và $\nabla \times W = A$ không.

$$\nabla.W = \alpha\left[(r.B)(\nabla.r) + r.\nabla(r.B)\right] + \beta\left[r^2(\nabla.B) + B.\nabla(r^2)\right]$$

$$\nabla r = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1+1+1 = 3; \nabla(r.B) = r \times (\nabla \times B) + B \times (\nabla \times r) + (r.\nabla)B + (B.\nabla)r;$$ [redacted]

[redacted] $\nabla \times$ [redacted] = [redacted]

$$\nabla(r.B) = (B.\nabla)r = \left(B_x \frac{\partial}{\partial x} + B_i \frac{\partial}{\partial y} + B_z \frac{\partial}{\partial z}\right)(x\hat{x} + y\hat{y} + z\hat{z}) = B_x\hat{x} + B_y\hat{y} + B_z\hat{z} = B;$$

vì vậy

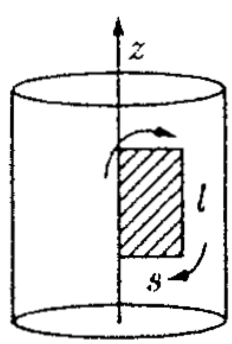
$$\nabla(r^2) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)(x^2 + y^2 + z^2) = 2x\hat{x} + 2y\hat{y} + 2z\hat{z} = 2r.$$

Vì vậy

$\nabla.W = \alpha[3(r.B) + (r.B)] + \beta[0 + 2(r.B)] = 2(r.B)(2\alpha + \beta)$, sẽ bằng không nếu $2\alpha + \beta = 0$

$$\nabla \times W = \alpha\left[(r.B)(\nabla \times r) - r \times \nabla(r.B)\right] + \beta\left[r^2(\nabla \times B) - B \times \nabla(r^2)\right] = \alpha[0 - (r \times B)] + \beta[0 - 2(B \times r)]$$

$$= -(r \times B)(\alpha - 2\beta) = -\frac{1}{2}(r \times B)$$



Chương10

THẾ VÀ TRƯỜNG

Bài tập 10.1

$$\square^2 V+\frac{\partial L}{\partial t}=\nabla^2 V-\mu_0\varepsilon_0\frac{\partial^2 V}{\partial t^2}+\frac{\partial}{\partial t}\left(\nabla.A\right)+\mu_0\varepsilon_0\frac{\partial^2 V}{\partial t^2}=\nabla^2 V+\frac{\partial}{\partial t}\left(\nabla.A\right)=-\frac{1}{\varepsilon_0}\rho$$

$$\square^2 A-\nabla L=\nabla^2 A-\frac{\partial^2 A}{\partial t^2}-\nabla\left(\left(\nabla.A\right)+\mu_0\varepsilon_0\frac{\partial V}{\partial t}\right)=\mu_0 J$$

Bài tập 10.2

(a) $\mathrm{W}=1/2\int\left(\varepsilon_0 E^2+\frac{1}{\mu_0}B^2\right)d\tau.At.t_1=d/c, x\geq d=ct_1,$ do đó E = 0, B=0, và do đó $\boxed{\mathrm{W}\left(t_1\right)=0}$

Tại $T_2=\left(d+h\right)/c, ct_2=d+h:$

$$E=-\frac{\mu_0\alpha}{2}\left(d+h-x\right)\hat{z}, B=\frac{1}{c}\frac{\mu_0\alpha}{2}\left(d+h-x\right)\hat{y},$$

Do đó: $B^2=\frac{1}{c^2}E^2,$ và $\left(\varepsilon_0 E^2+\frac{1}{\mu_0}B^2\right)=\varepsilon_0\left(E^2+\frac{1}{\mu_0\varepsilon_0}\frac{1}{c^2}E^2\right)=2\varepsilon_0 E^2$

Vì thế:

$$\mathrm{W}\left(t_2\right)=\frac{1}{2}\left(2\varepsilon_0\right)\left(\frac{\mu_0^2\alpha^2}{4}\right)\int_d^{d+h}\left(d+h-x\right)^2 dx\left(l\omega\right)=\frac{\varepsilon_0\mu_0^2\alpha^2 l\omega}{4}\left[-\frac{\left(d+h-x\right)^3}{3}\right]_d^{d+h}$$

$$=\boxed{\frac{\varepsilon_0\mu_0^2\alpha^2 l\omega h^3}{12}}$$

(b) $S\left(x\right)=\frac{1}{\mu_0}\left(\mathbf{B}\times\mathbf{E}\right)=\frac{1}{\mu_0 c}E^2\left[-\hat{z}\times\left(\pm\hat{y}\right)\right]=\pm\frac{1}{\mu_0 c}E^2\hat{x}=\boxed{\pm\frac{\mu_0\alpha^2}{4c}\left(ct-|x|\right)^2\hat{x}}$

(Dấu cộng khi $x\rangle 0$, như ở đây). Khi $|x|\rangle ct, S=0$

Vì vậy năng lượng trên một đơn vị thời gian vào hộp trong khoảng thời gian này là:

$$\frac{d\mathrm{W}}{dt}=P=\int S\left(d\right).da=\boxed{\frac{\mu_0\alpha^2 l\omega}{4c}\left(ct-d\right)^2.}$$

Chú ý rằng không có năng lượng chảy lên phía trên, bởi vì $\mathbf{S}(d+h)=0$

(c) $W = \int_{t_1}^{t_2} P dt = \frac{\mu_0 \alpha^2 l \omega}{4c} \int_{d/x}^{(d+h)/c} (ct-d)^2 = \frac{\mu_0 \alpha^2 l \omega}{4c} \left[\frac{(ct-d)^3}{3c} \right]_{d/c}^{(d+h)/c} = \boxed{\frac{\mu_0 \alpha^2 l \omega h^3}{12c}}$

Bởi vì $1/c^2 = \mu_0 \varepsilon_0$, kết quả này phù hợp với kết quả ở câu a.

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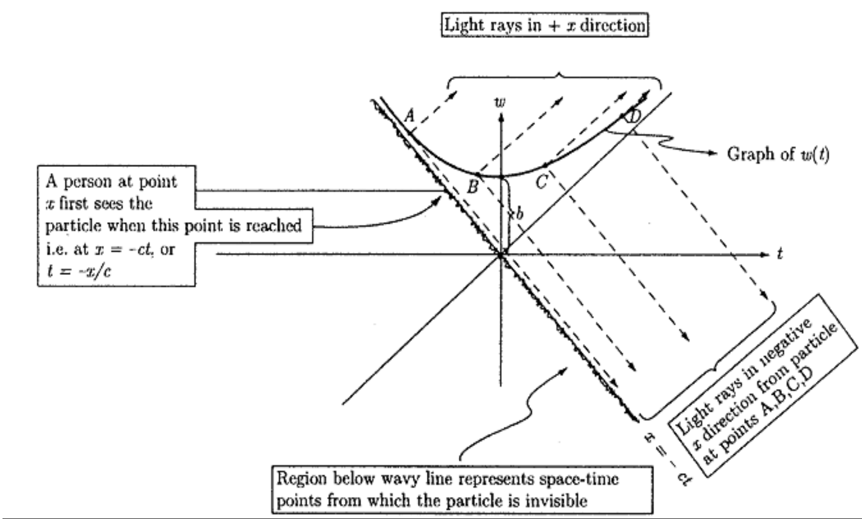

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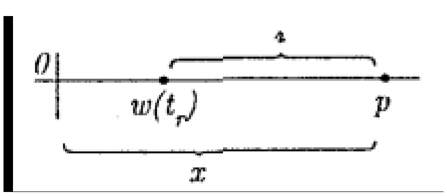

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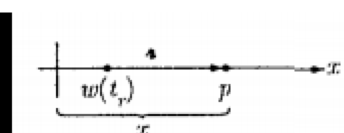

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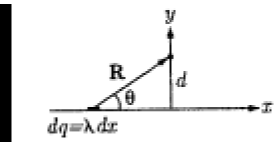

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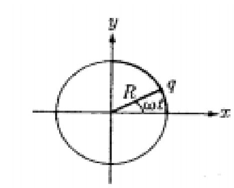

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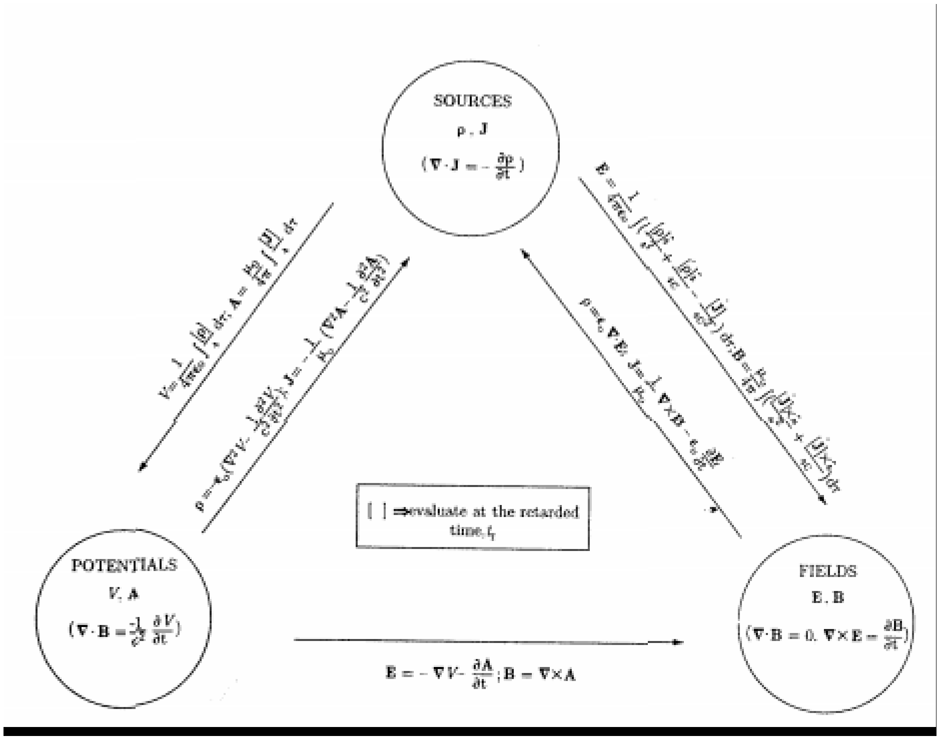

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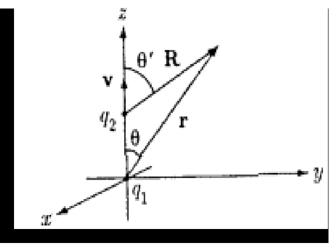

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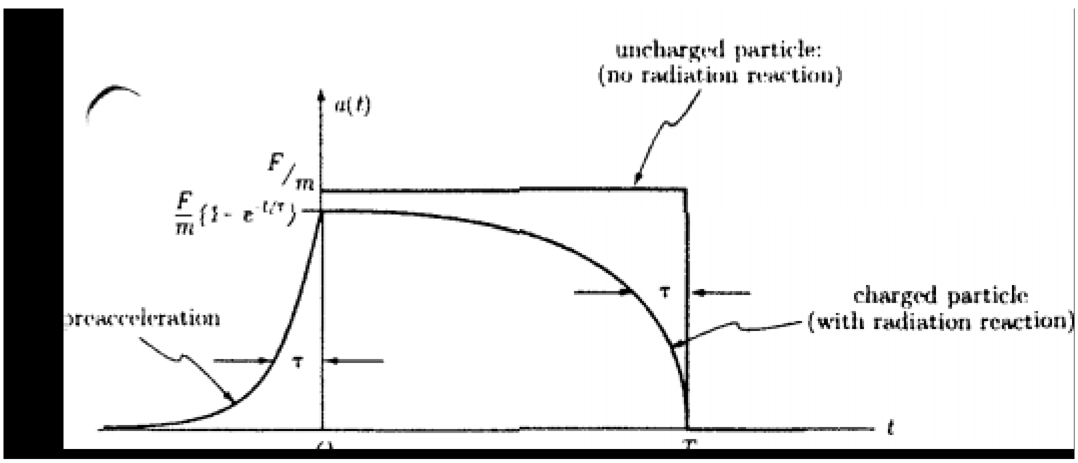

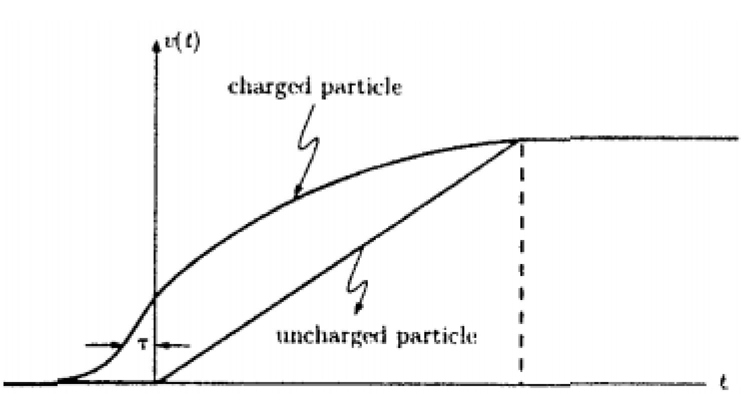

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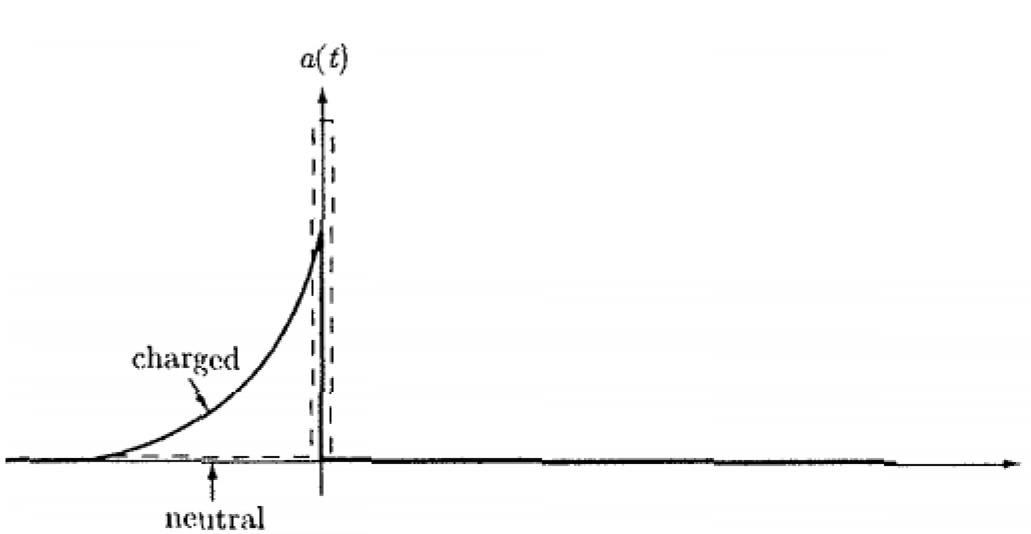

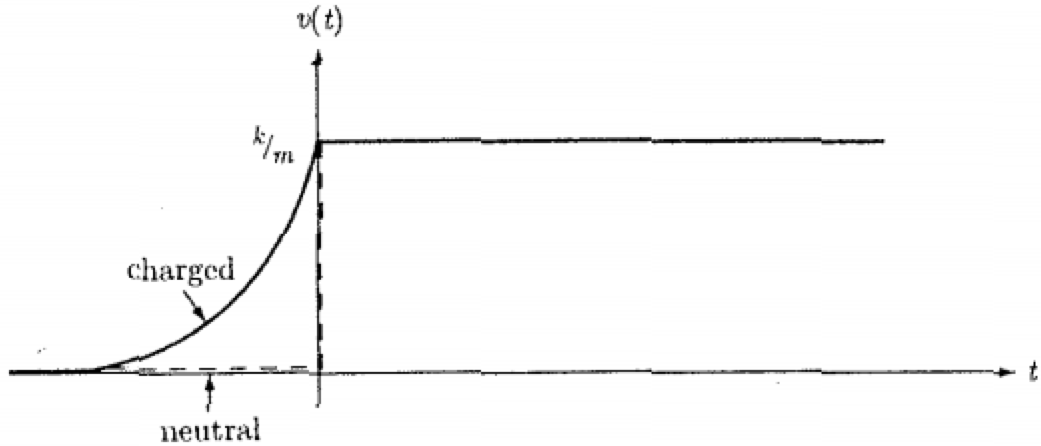

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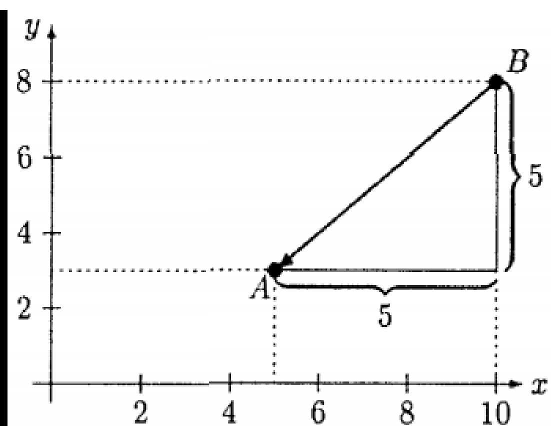

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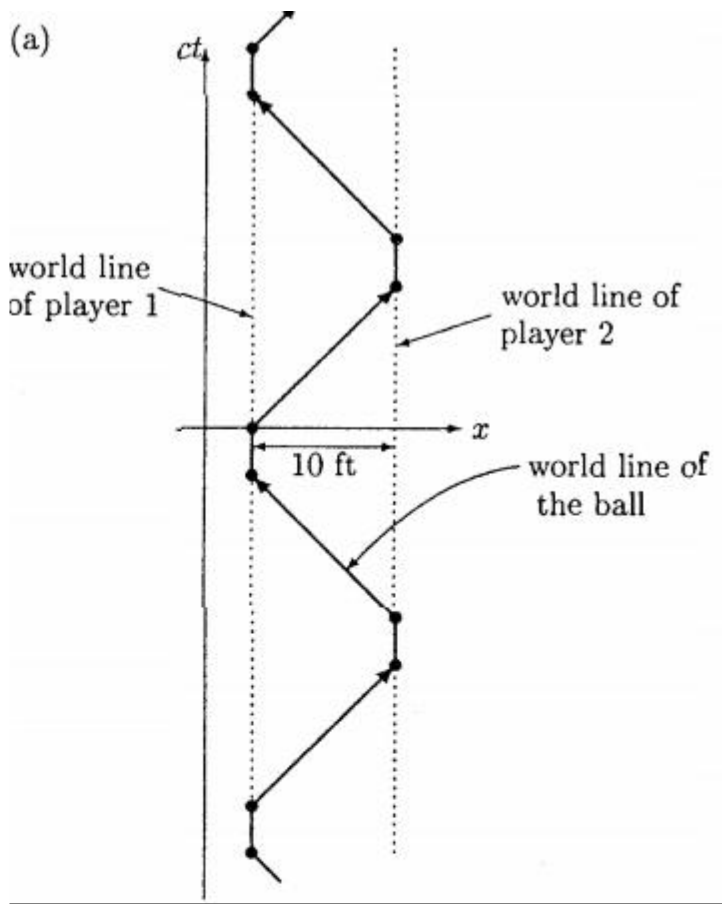
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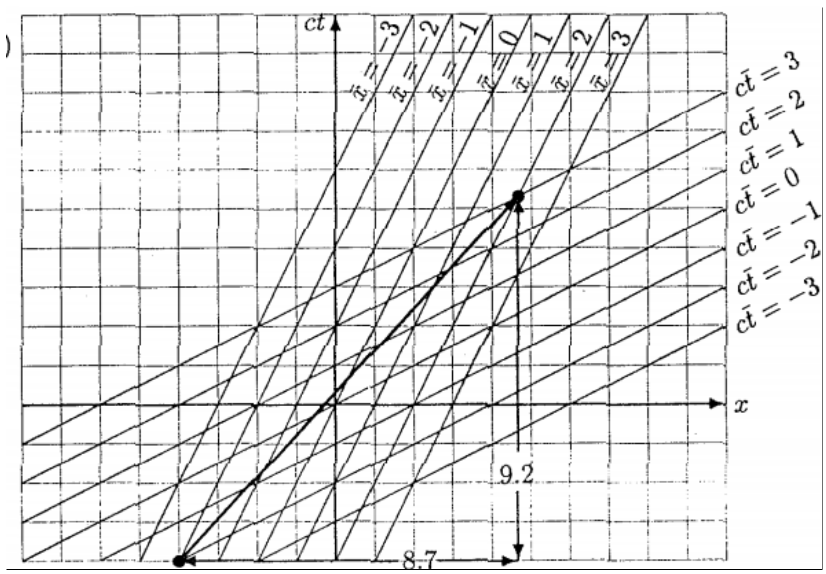

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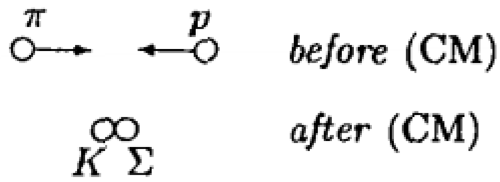

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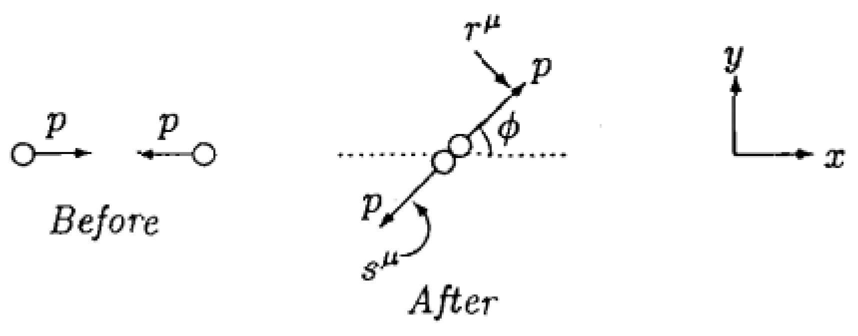

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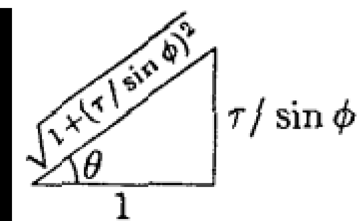

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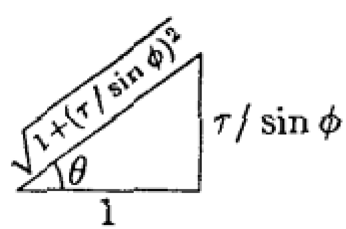

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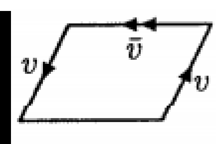

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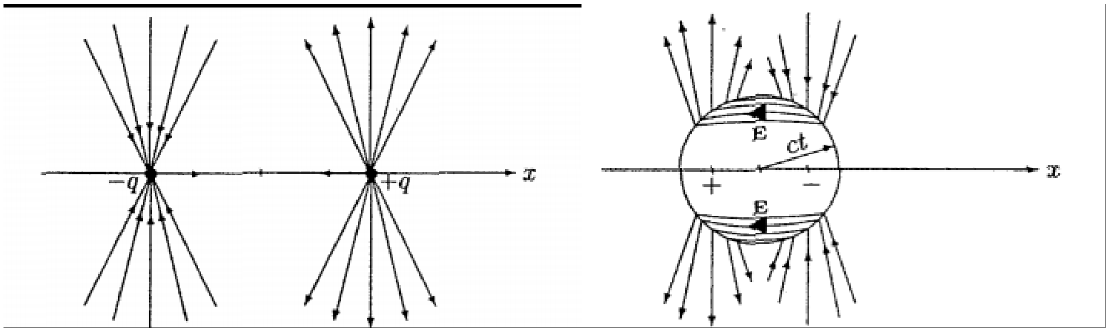

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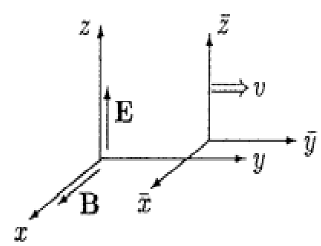

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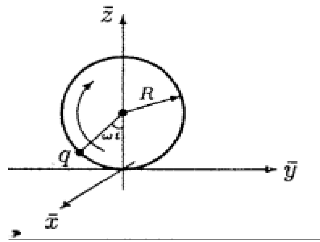

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